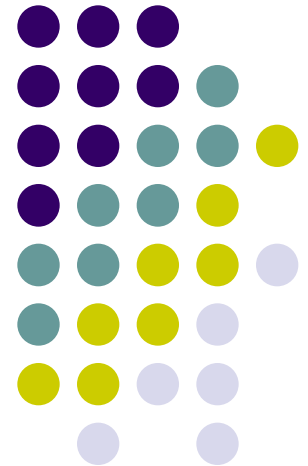


# Analog computers

Francis Massen  
Computarium LCD

[francis.massen@education.lu](mailto:francis.massen@education.lu)

<http://computarium.lcd.lu>



# Index



- Definition of an analog computer
- Mechanical analog calculators & computers
- Electronic analog computers
- Demonstrations
- The future. Literature and links

# Definition



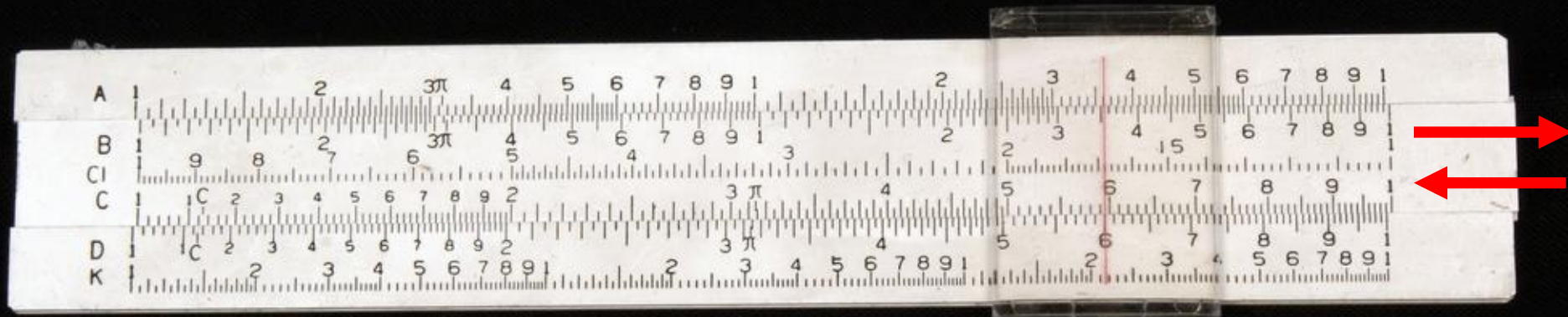
- An analog computer works in a **continuous** manner (a digital computer functions in discrete steps).
- An analog computer uses a model which behaves in a similar way (= in an “analog” way) to the problem to be solved.

The oldest analog computers were purely mechanical systems, later systems were electronic devices.

# Mechanical analog computer

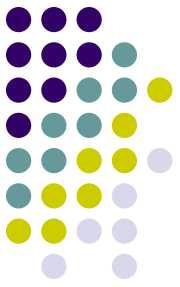


Example: Sliderule

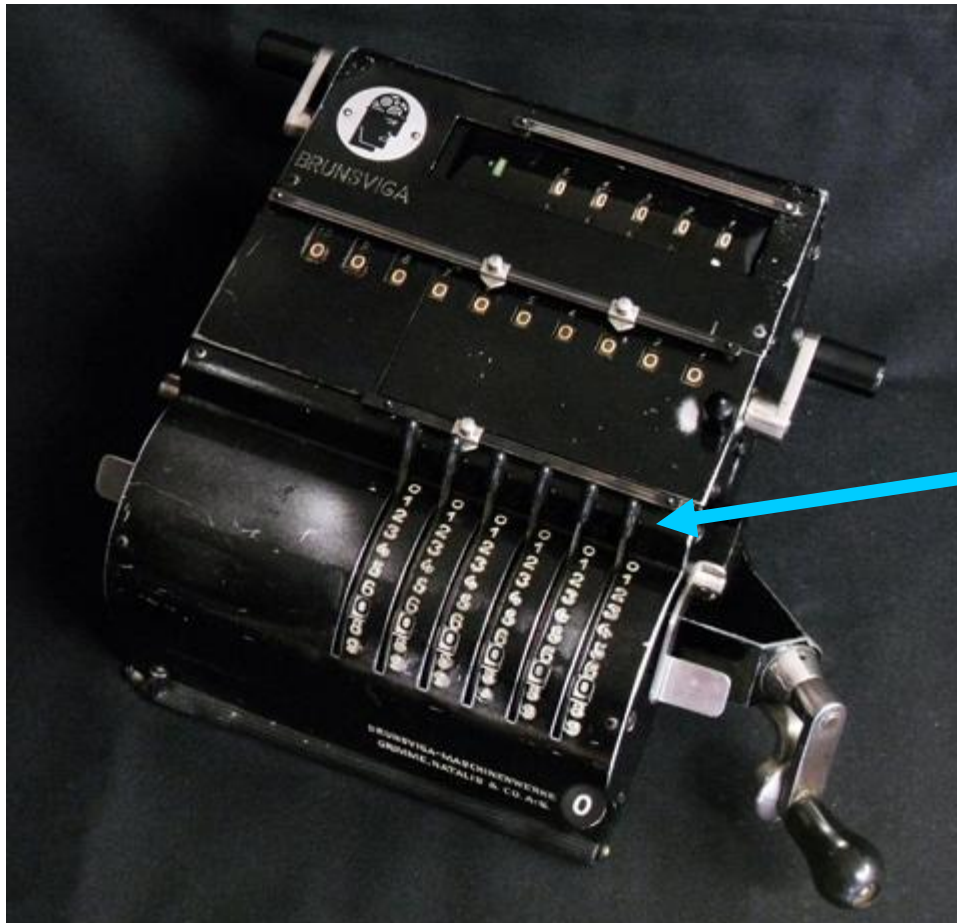


Central ruler moves continuously.

# Mechanical digital computer



Example: Brunsviga calculator (mod. 10, 1925)



Slider can take only fixed positions 0...9



# Mechanical analog computer

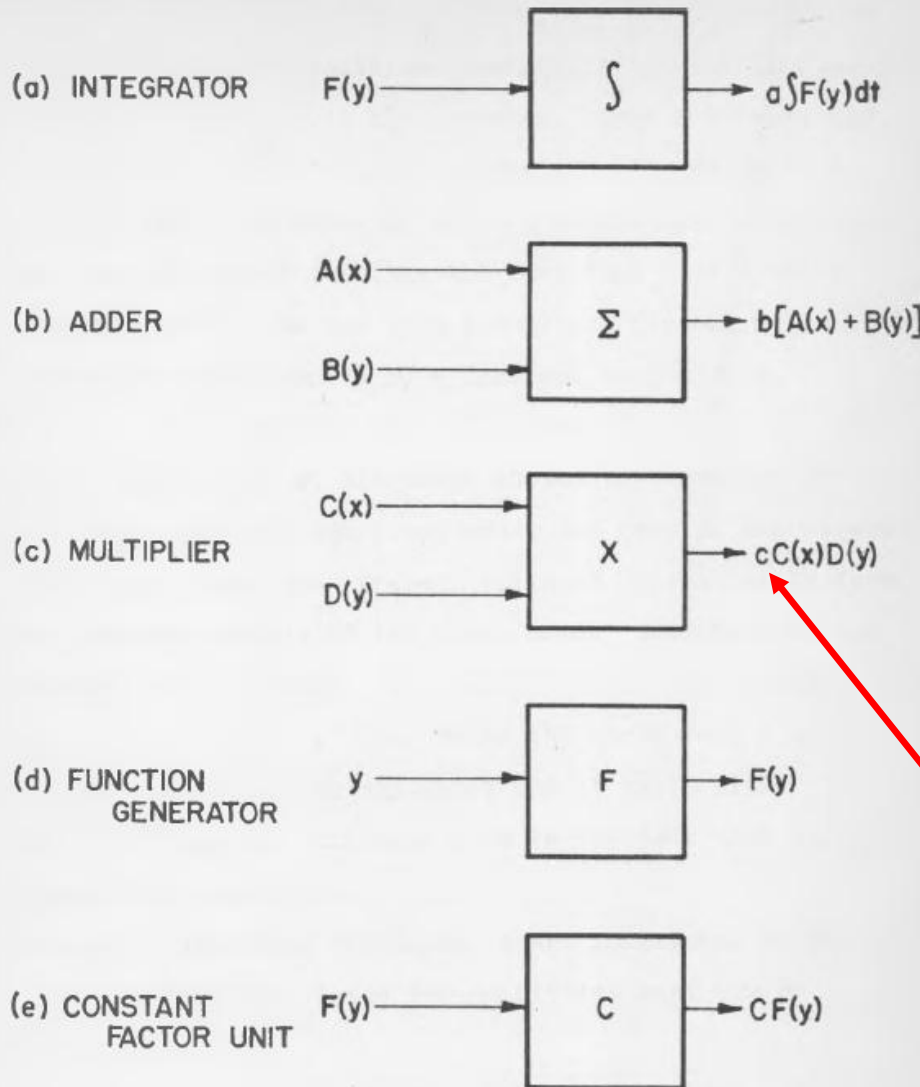


Antikythera, 78 BC (astrolabe)  
Oldest analog computer known:  
at least 30 gears!  
(found in 1900 by sponge divers)



Several rebuilds since 1978  
(Antikythera Mechanism Research  
project)

# Mechanical analog computer

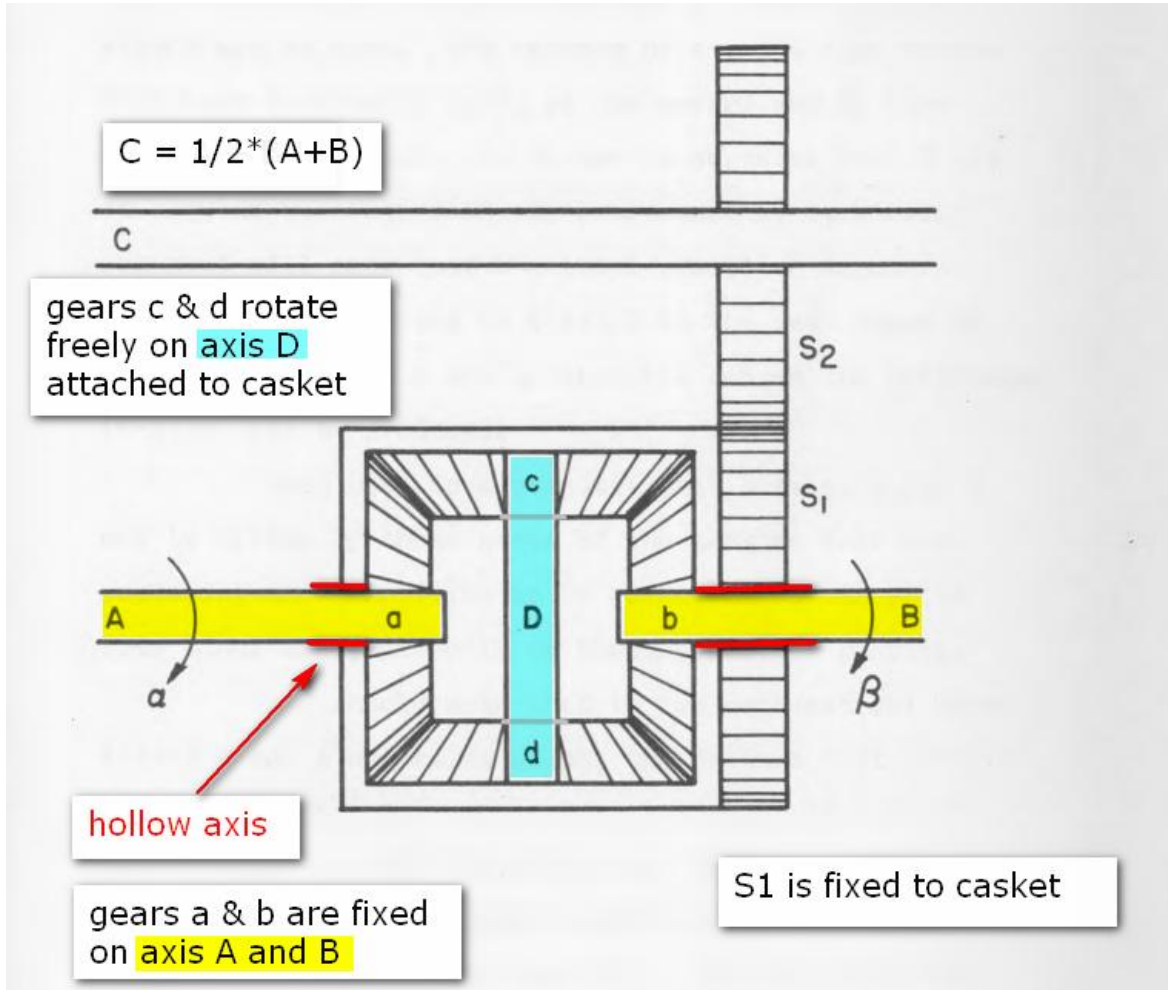


Only 5 functions are needed to solve (nearly) any problem described by ordinary differential equations

F.J. Miller:  
Theory of Mathematical Machines,  
1947

Output is proportional to the mathematical operator !

# Example: Adder [Thesis J.E. Kasper, 1955]



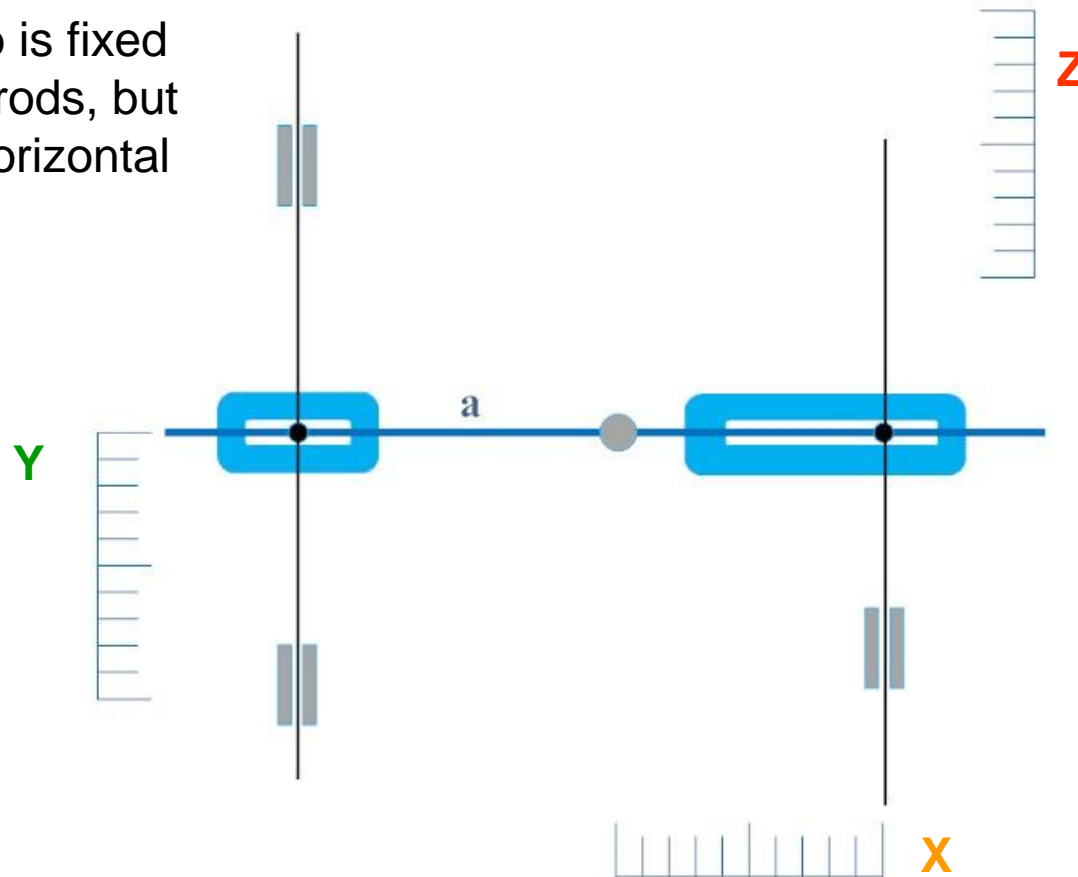


# Mechanical analog computer



## Example: Multiplier [Mathematische Instrumente, Meyer zur Capellen, 1941]

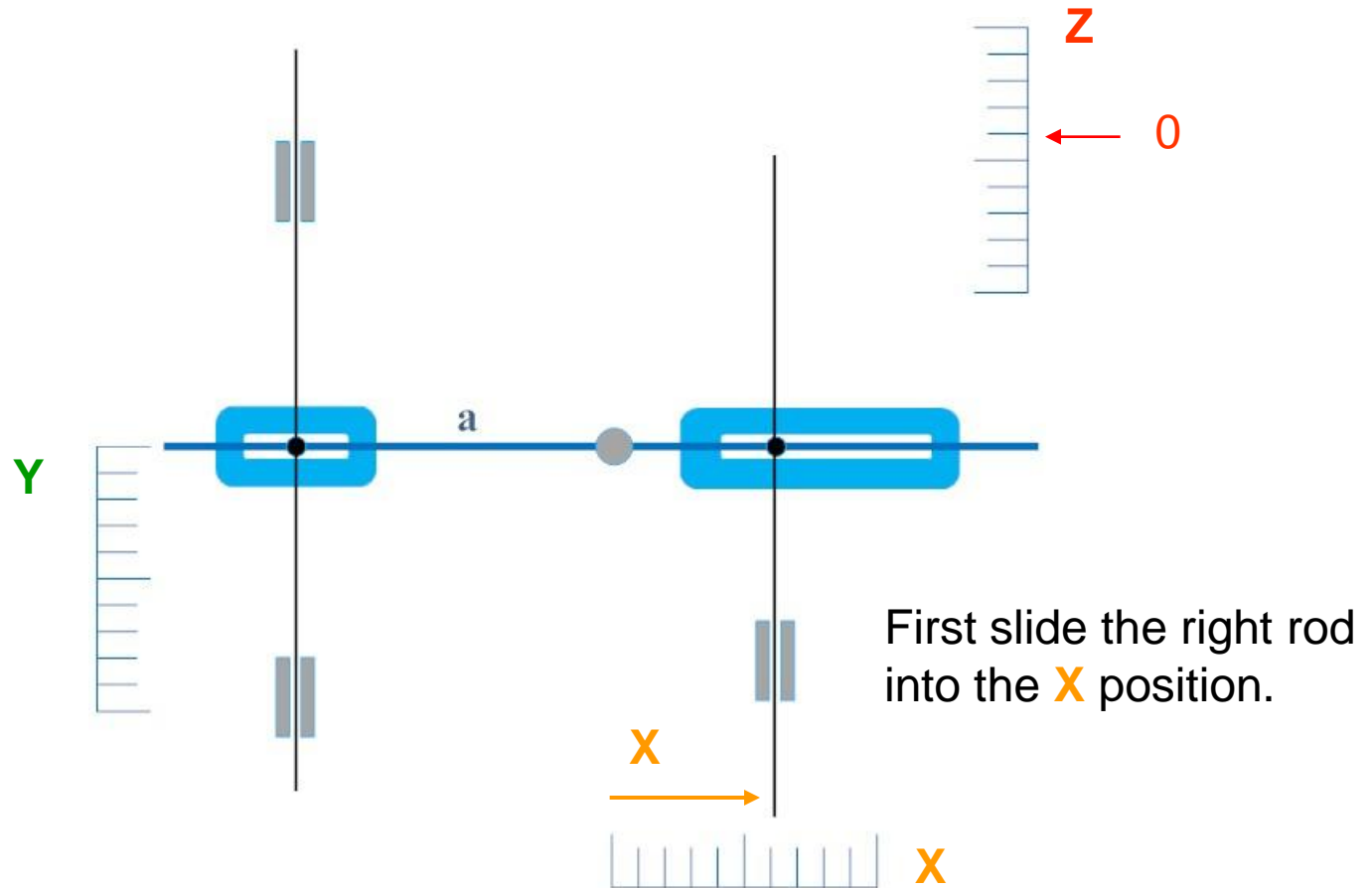
The black knob is fixed on the vertical rods, but slides on the horizontal level.



# Mechanical analog computer



Example: Multiplier [Mathematische Instrumente, Meyer zur Capellen, 1941]

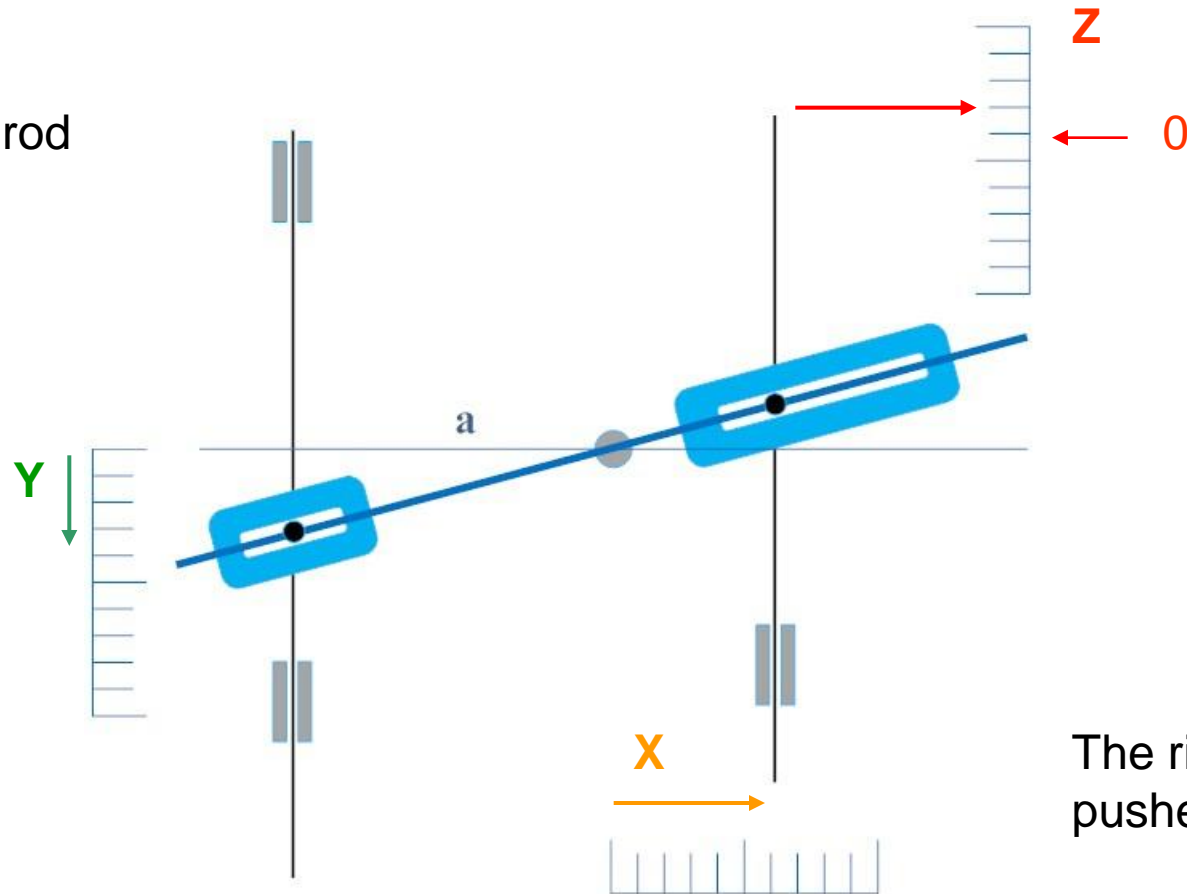


# Mechanical analog computer



## Example: Multiplier [Mathematische Instrumente, Meyer zur Capellen, 1941]

Now push the left rod down into the **Y** position

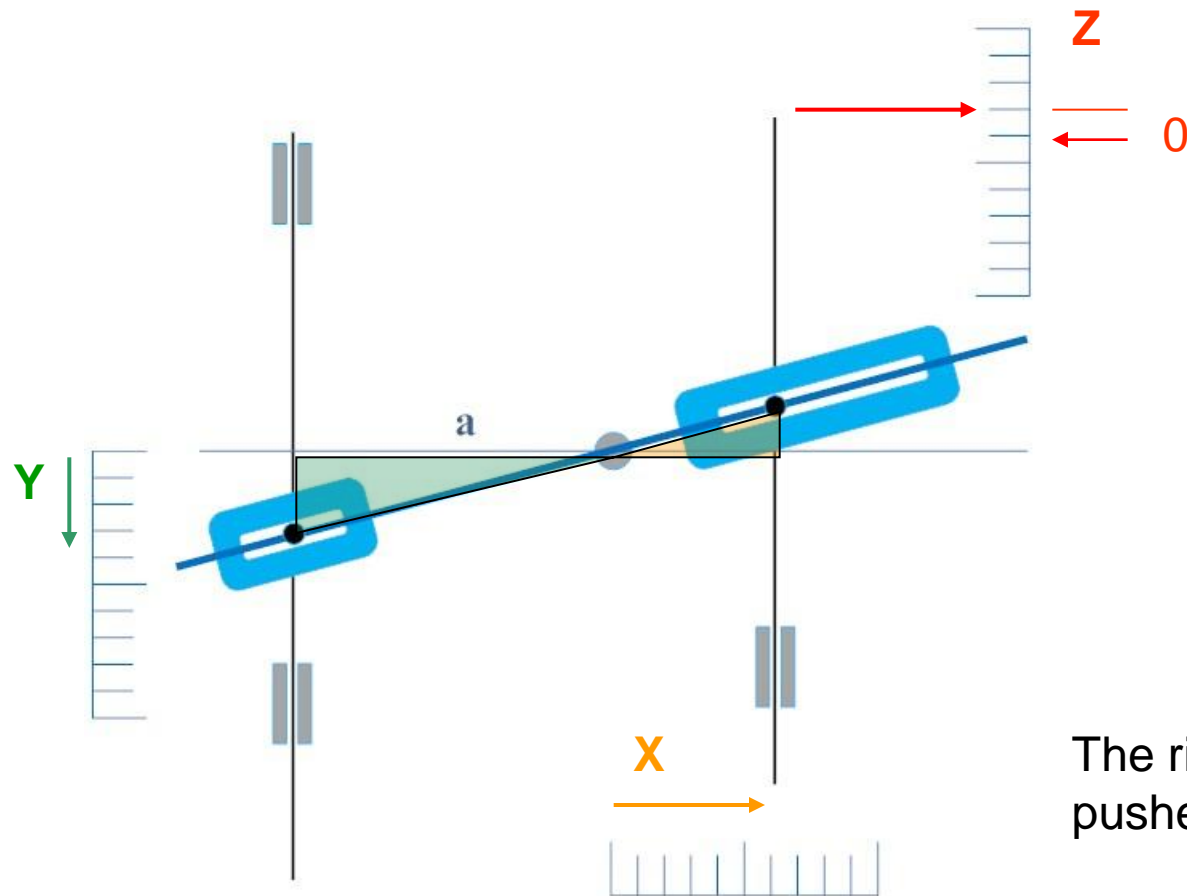


The right rod is pushed up by **Z**

# Mechanical analog computer



Example: Multiplier [Mathematische Instrumente, Meyer zur Capellen, 1941]



The right rod is pushed up by **Z**

# Mechanical analog computer

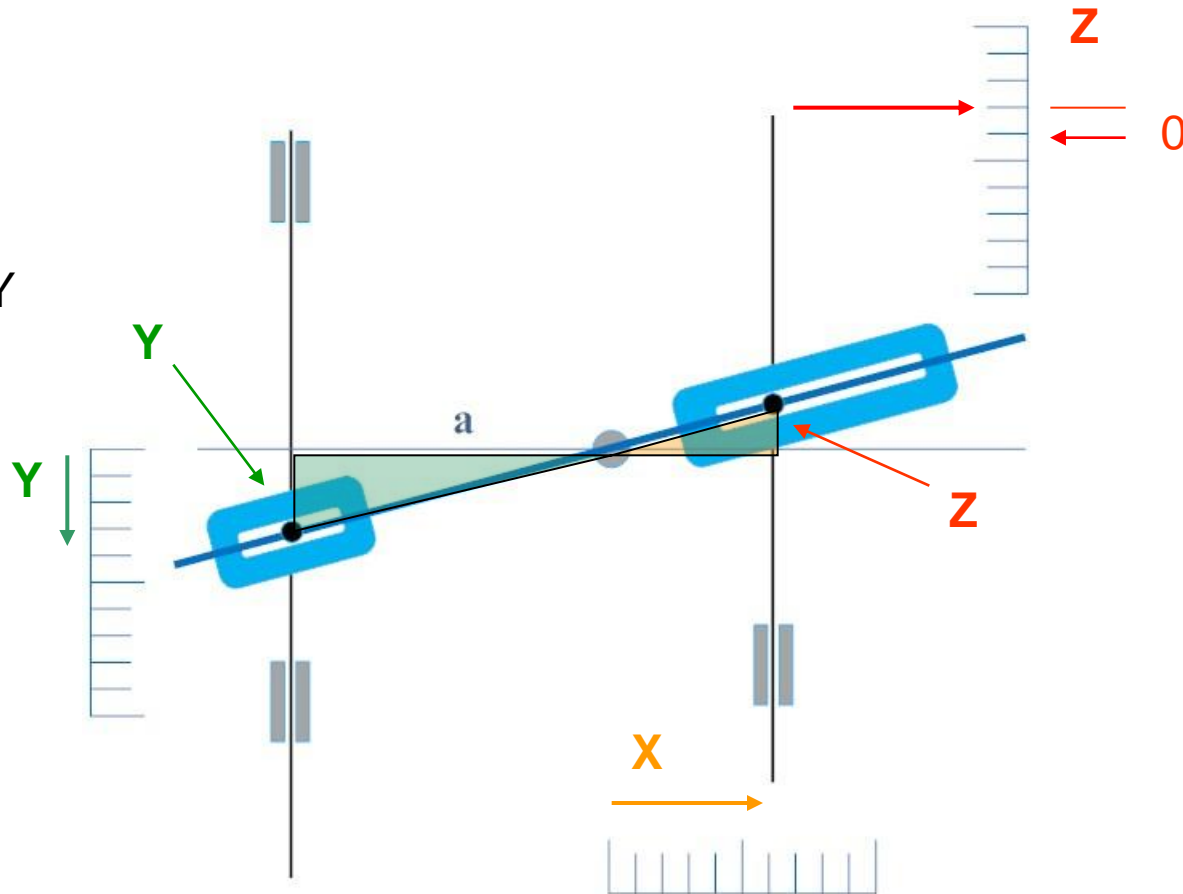


## Example: Multiplier [Mathematische Instrumente, Meyer zur Capellen, 1941]

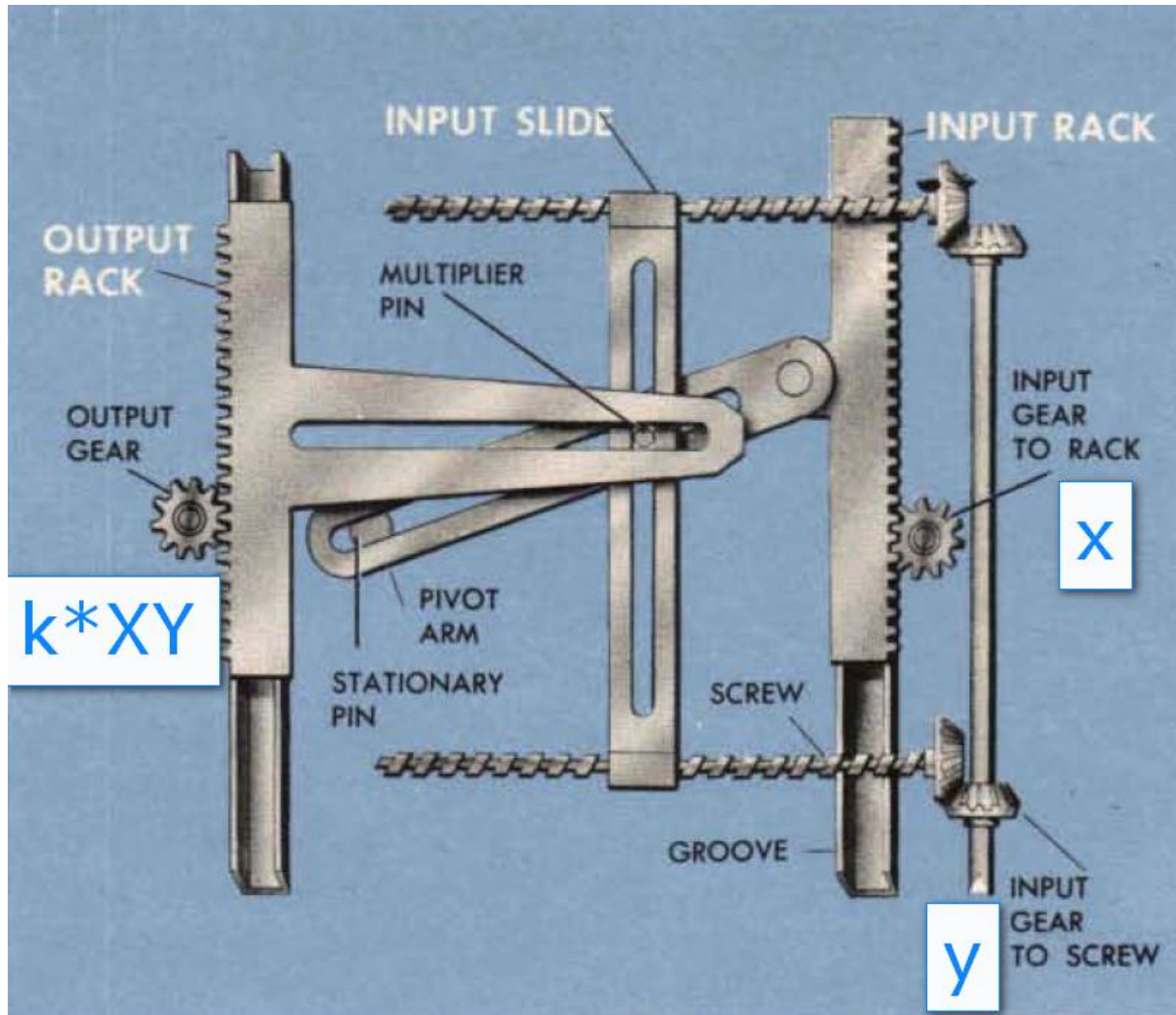
$$Z/X = Y/a$$

$$Z = 1/a \cdot (X \cdot Y)$$

$Z$  = proport. to  $X \cdot Y$



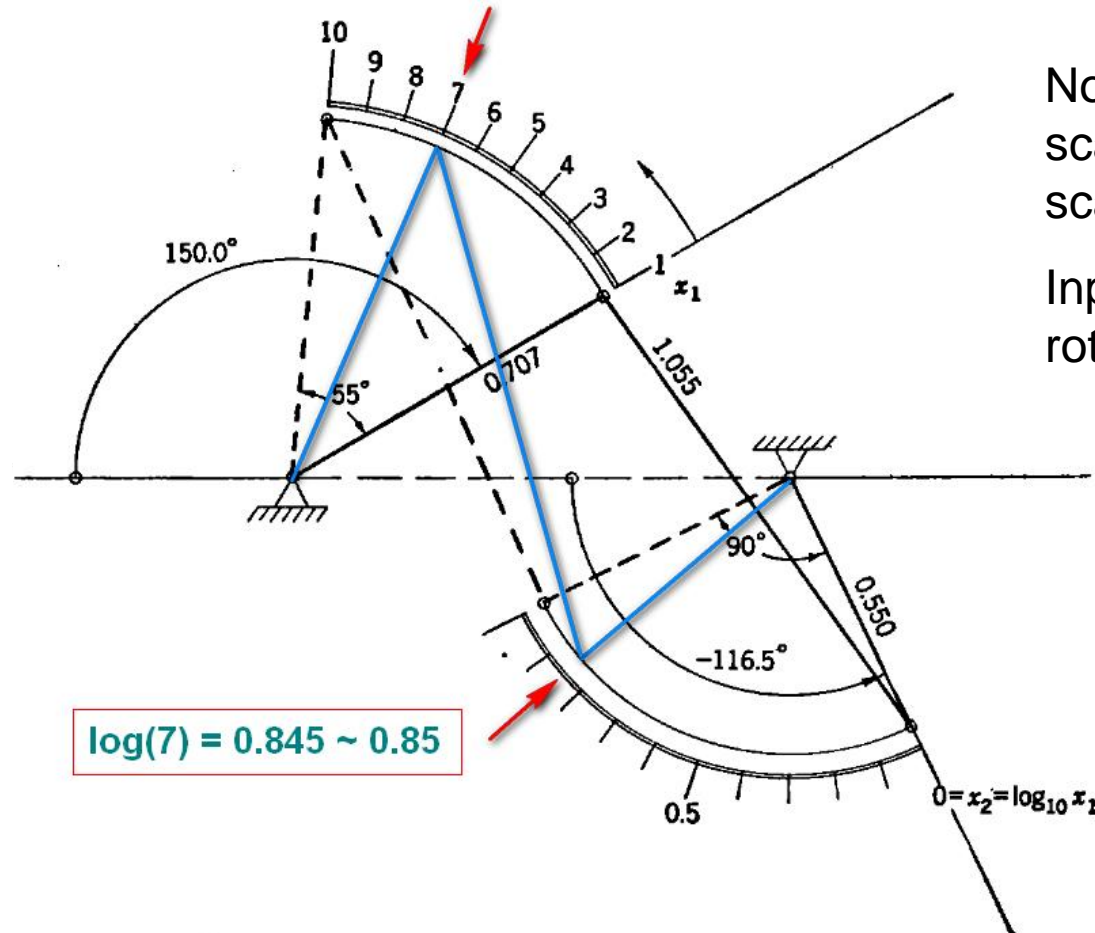
# Military screw multiplier



From ordnance  
pamphlet 1140  
Gun and Fire control  
(US Navy)  
<http://archive.hnsa.org/>



# Logarithm calculator



Note the regular scaling of both scales!

Input and output are rotations.

FIG. 5-28.—Approximate mechanization of  $x_2 = \log_{10} x_1$ .

# Sinus generator

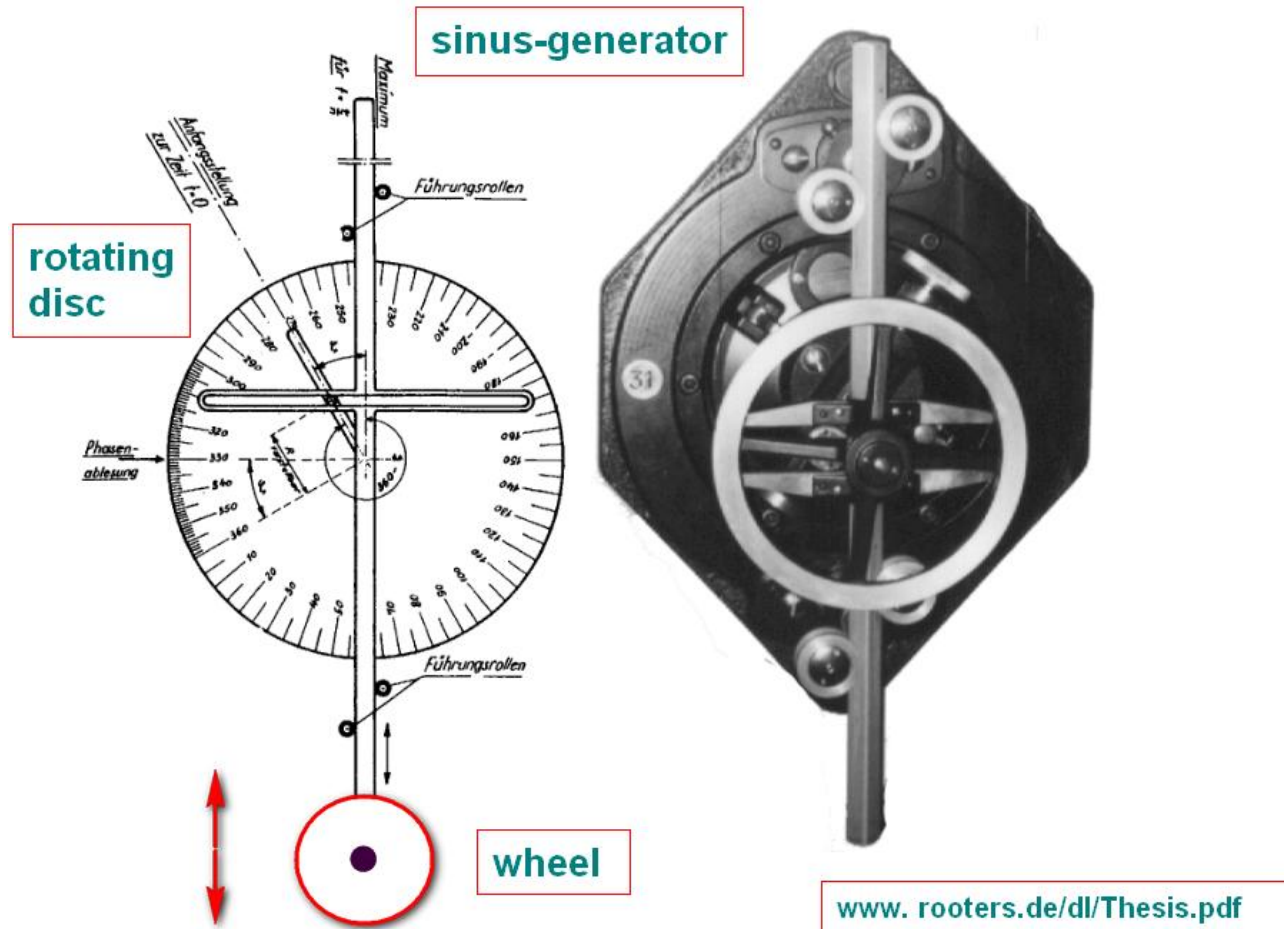


Abbildung 17: Generator zur Erzeugung harmonischer Schwingungen. Links Zeichnung (entnommen [Sag55] S. 33) und rechts Foto eines Generators der zweiten deutschen Gezeitenberechnungsmaschine

# Tide calculator (1)

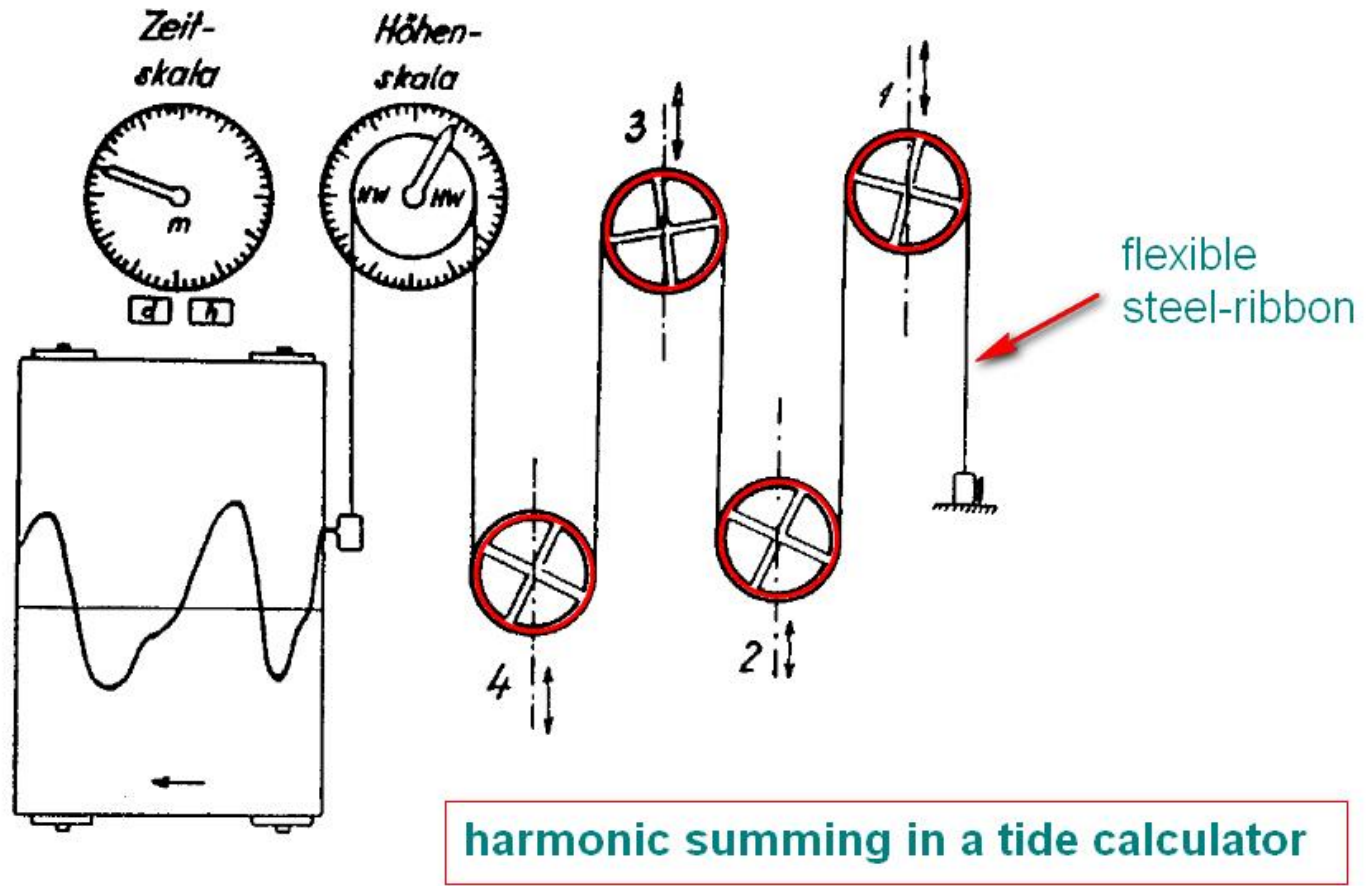
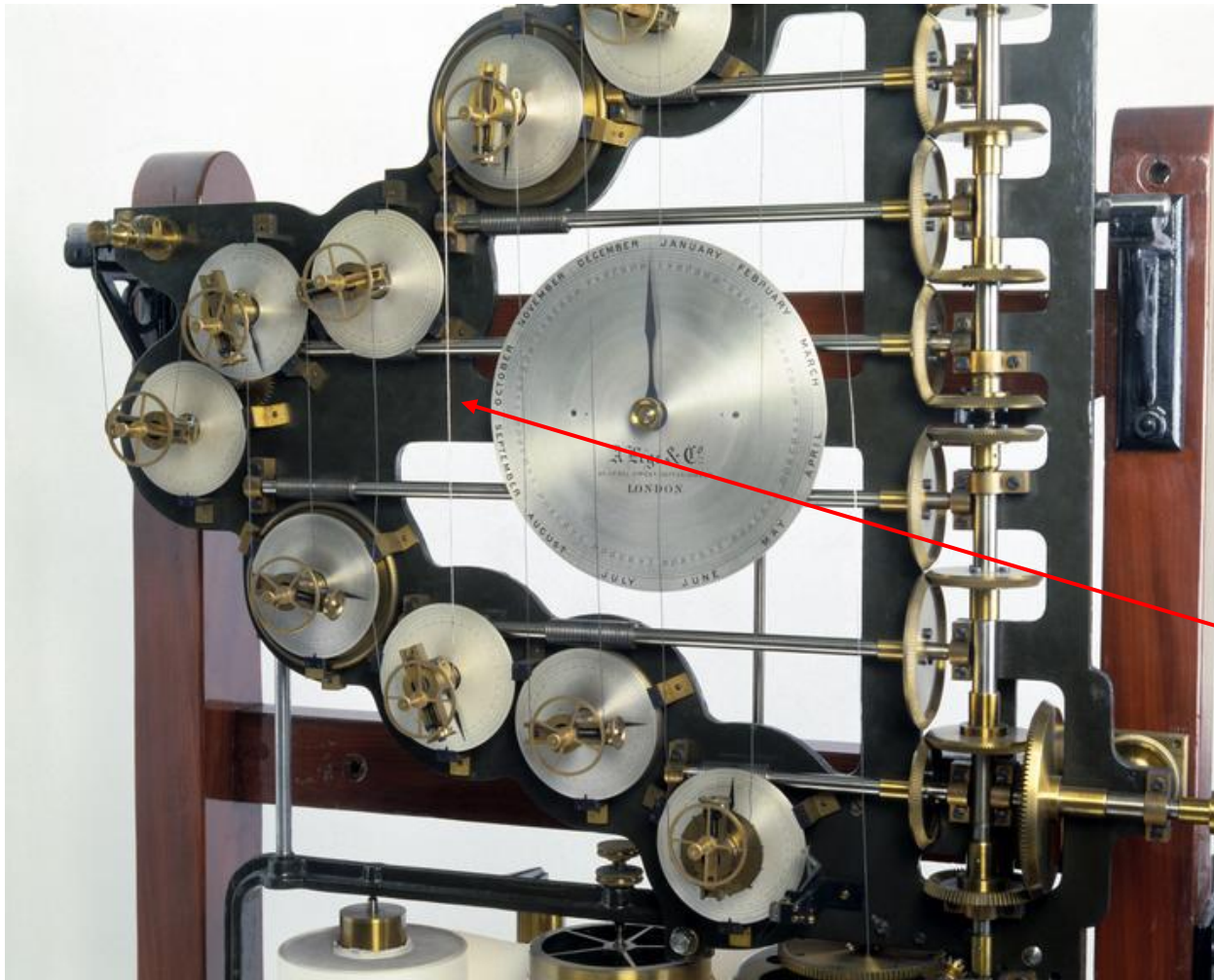


Abbildung 18: Mechanische Überlagerung harmonischer Schwingungen. (Entnommen [Sag55] S. 35)

# Tide calculator (2)



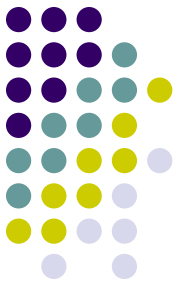
Kelvin tide  
predictor 1872:  
10 harmonics

Steel wire

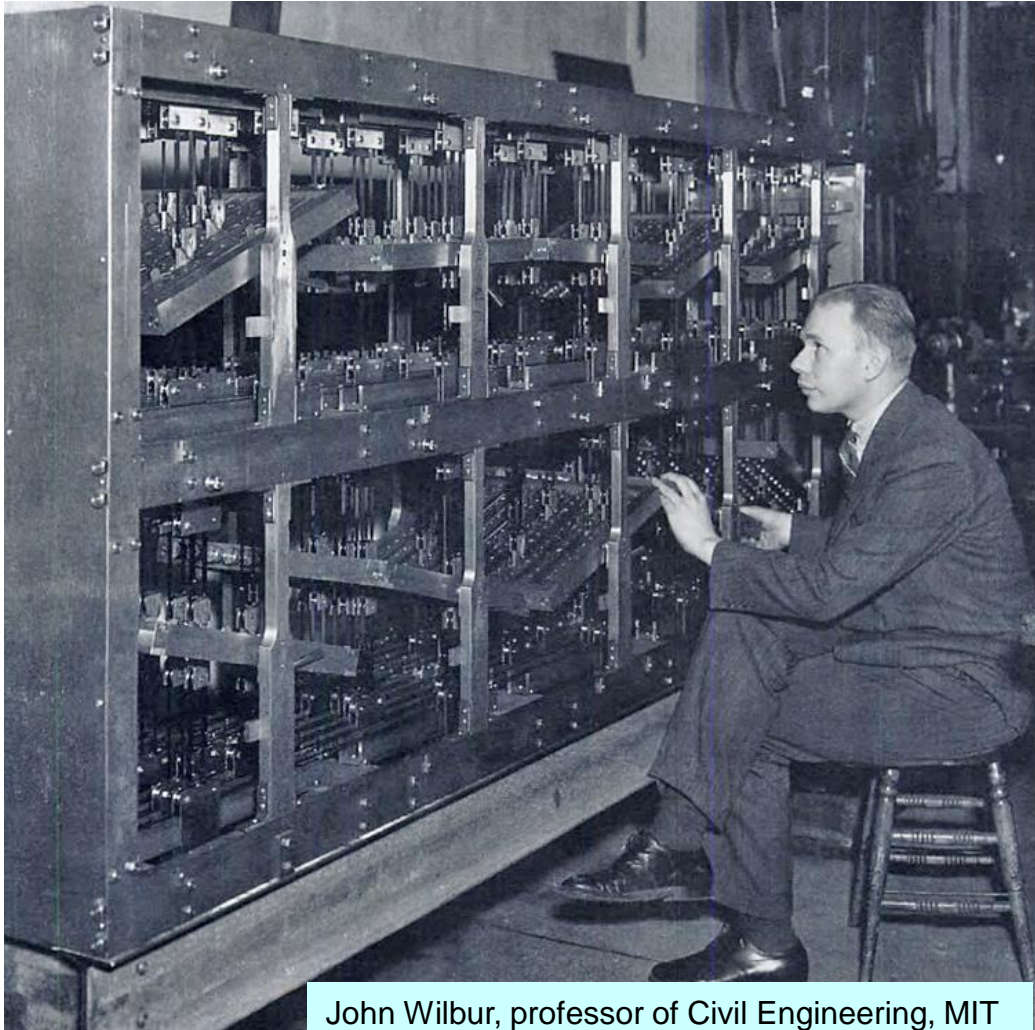
Lord Kelvin & Roberts: 1879, 20 harmonics



# Equation solver (1)



The Wilbur machine (MIT, 1936) solves systems of linear equations (13000 parts, > 1000 pulleys)



John Wilbur, professor of Civil Engineering, MIT

Built to solve systems of 9 linear equations:

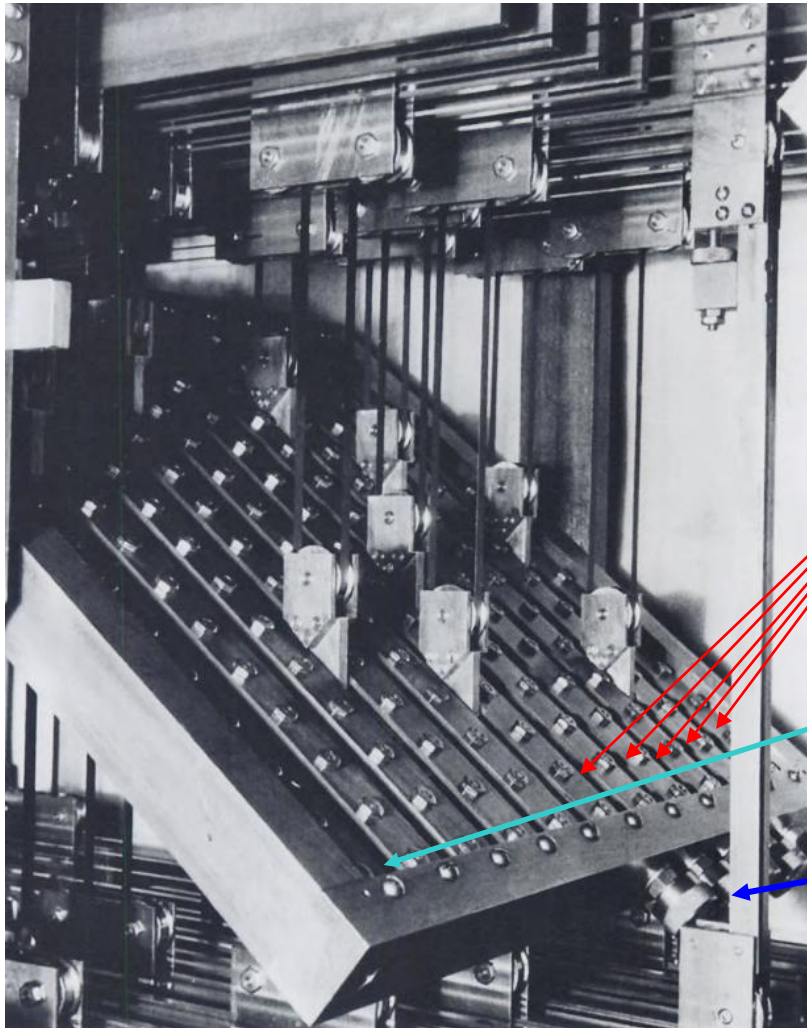
The sine of the angle of the slotted plate gives the solution of one variable.

(9 plates which 9 double-pulleys for every equation, 10<sup>th</sup> plate for the constants).

Only used once by Harvard economist W.W.Leontief to calculate an economic model which needed 450000 multiplications.

# Equation solver (2)

One plate of the Wilbur machine:



Wilbur machine: time to solve a system  
~1 to 3 hours. Without the machine  
Leontief's model would have needed  
2 years at 120 multiplications/hour

9 slots: one for  
each variable

10<sup>th</sup> slot to  
read  $\sin(\text{angle})$

Micrometer screw to  
move the pulley (set  
the coefficient)

*Figure 12. The detail shows one of the plates with some of its nearly 1000 pulleys leading the steel tape. Above and below some of them can be shifted horizontally. Notice the controlling knobs at the plate. (Photo MIT Museum)*



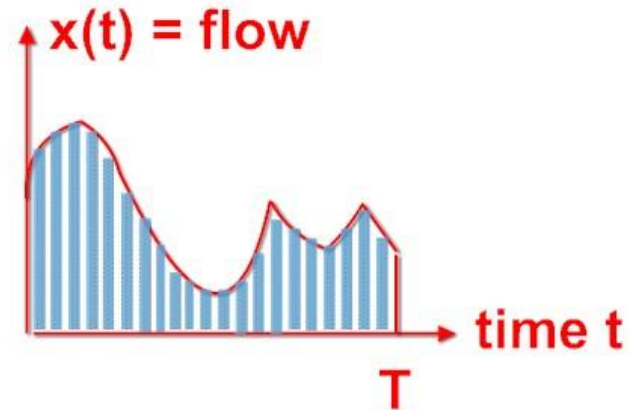
# What is an integral?



variable flow  $x(t)$  [l/s]



$V(t)$

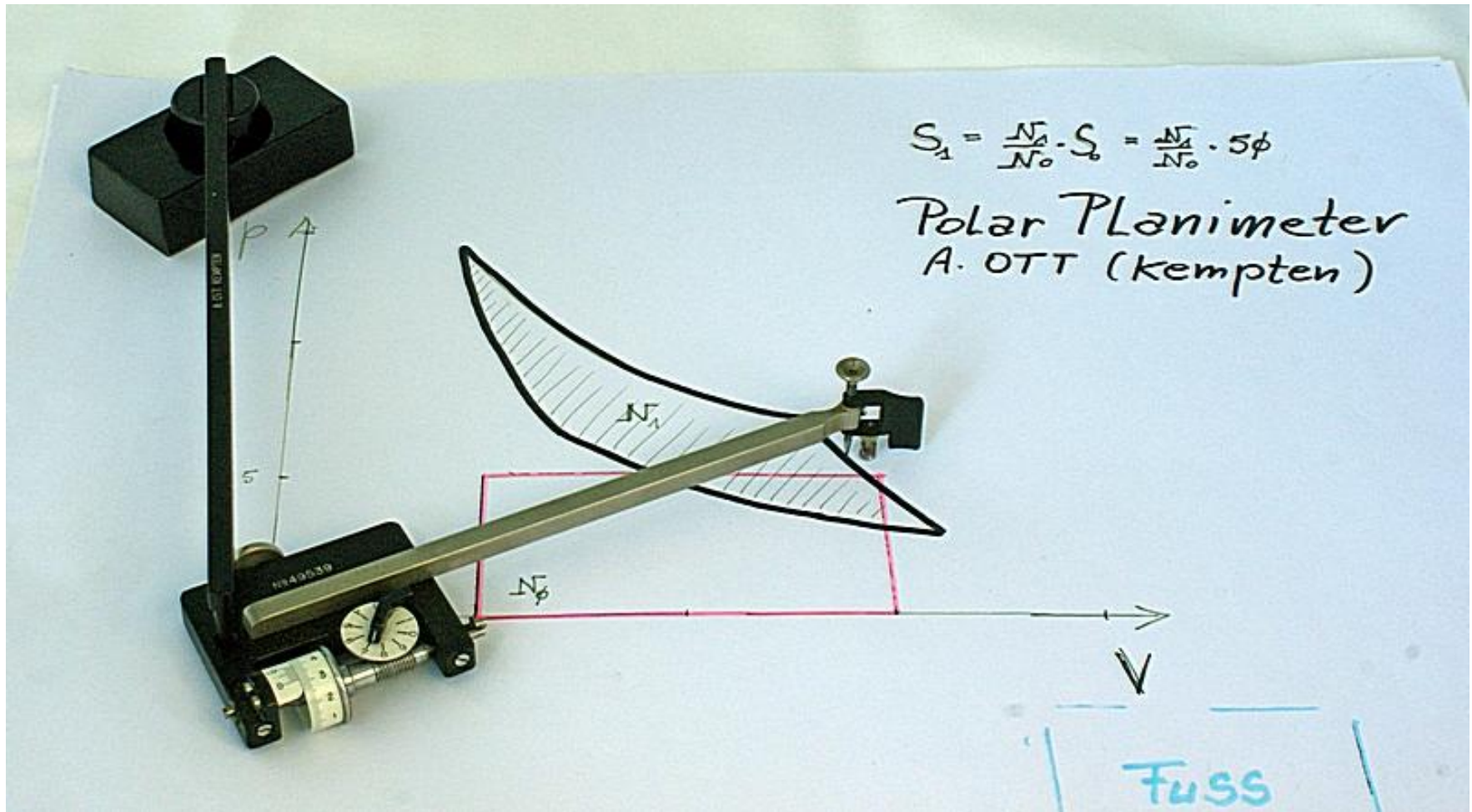
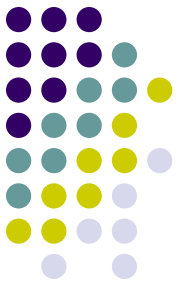


integral = area below curve  
= volume at time T

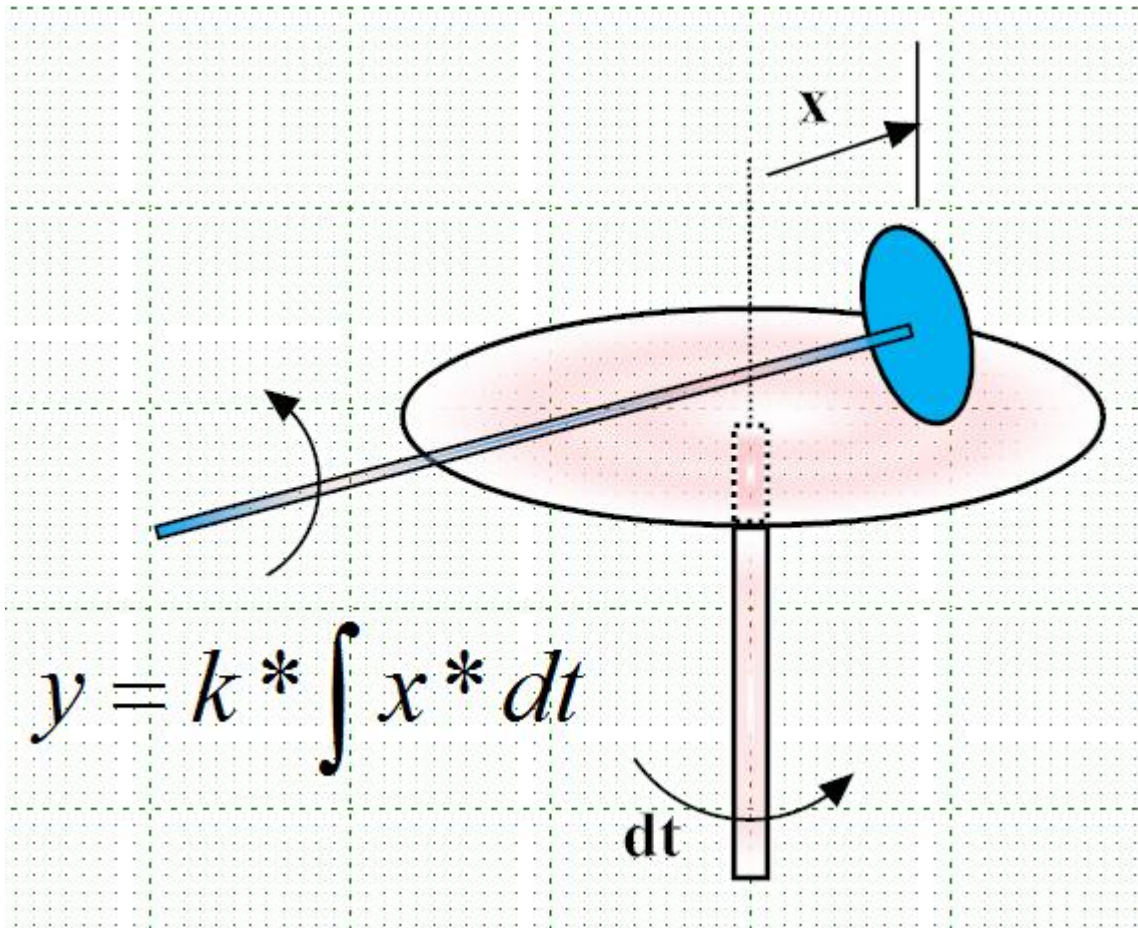
$$V = k * \int_0^T x(t) * dt$$

# Integrators: planimeter

Made from approx. 1850 - 1970



# Integrators: disc type

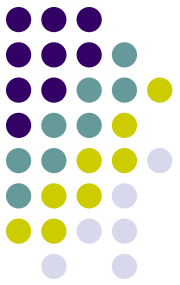
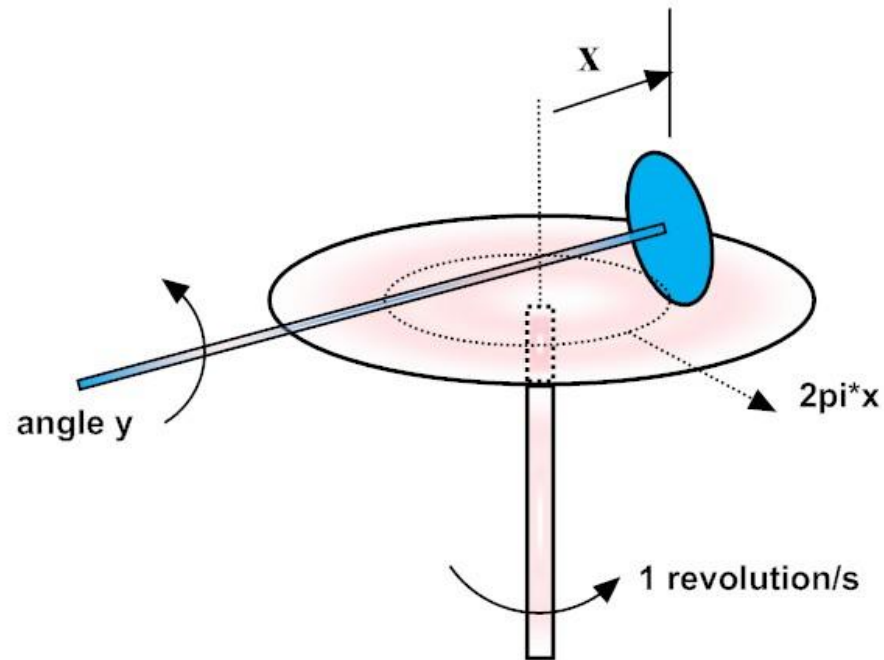


The Kelvin disc integrator

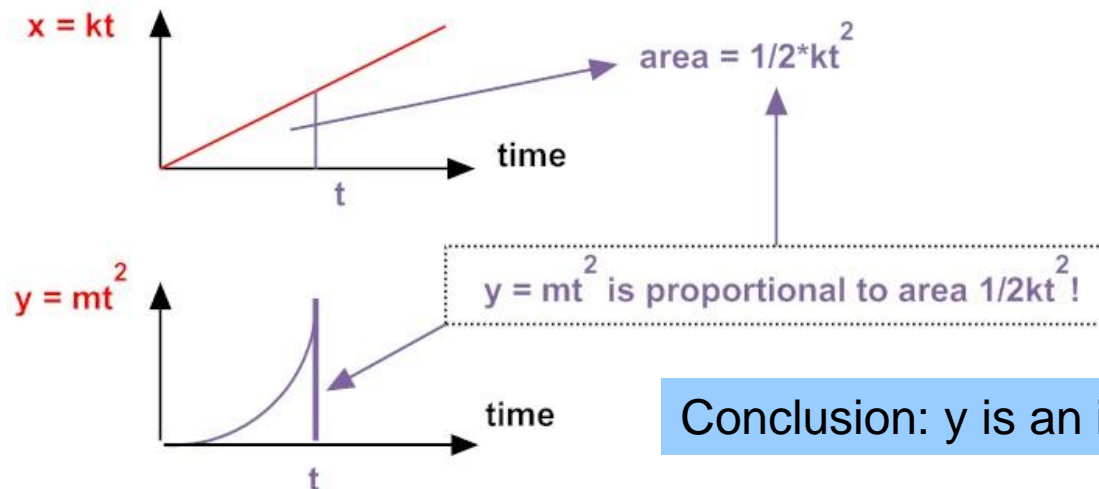
← usually roughened glass disc

If  $x$  increases linearly in time,  $y$  increases proportional to the square of time.

# “Proof”

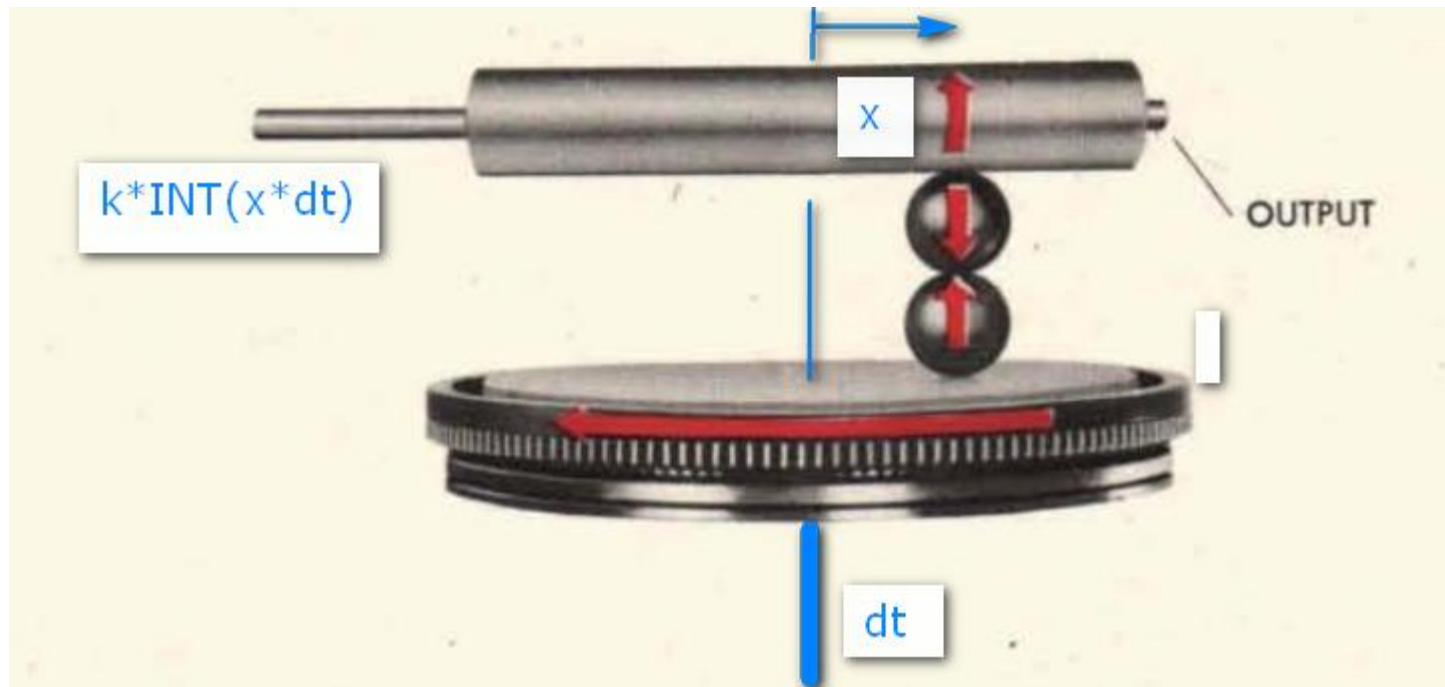


1. In 1 second angle  $y = (2\pi x / 2\pi r) * 2\pi = (x/r) * 2\pi$  [r = radius of small disk]
  2. In t seconds angle  $y = (x/r) * 2\pi t$
  3. If x increases linearly with t:  $x = k * t$  and  $y = kt/r * 2\pi t = 2\pi k/r * t^2 = m * t^2$
- so  $y = m * t^2$



Conclusion:  $y$  is an integral

# Integrator: roller type

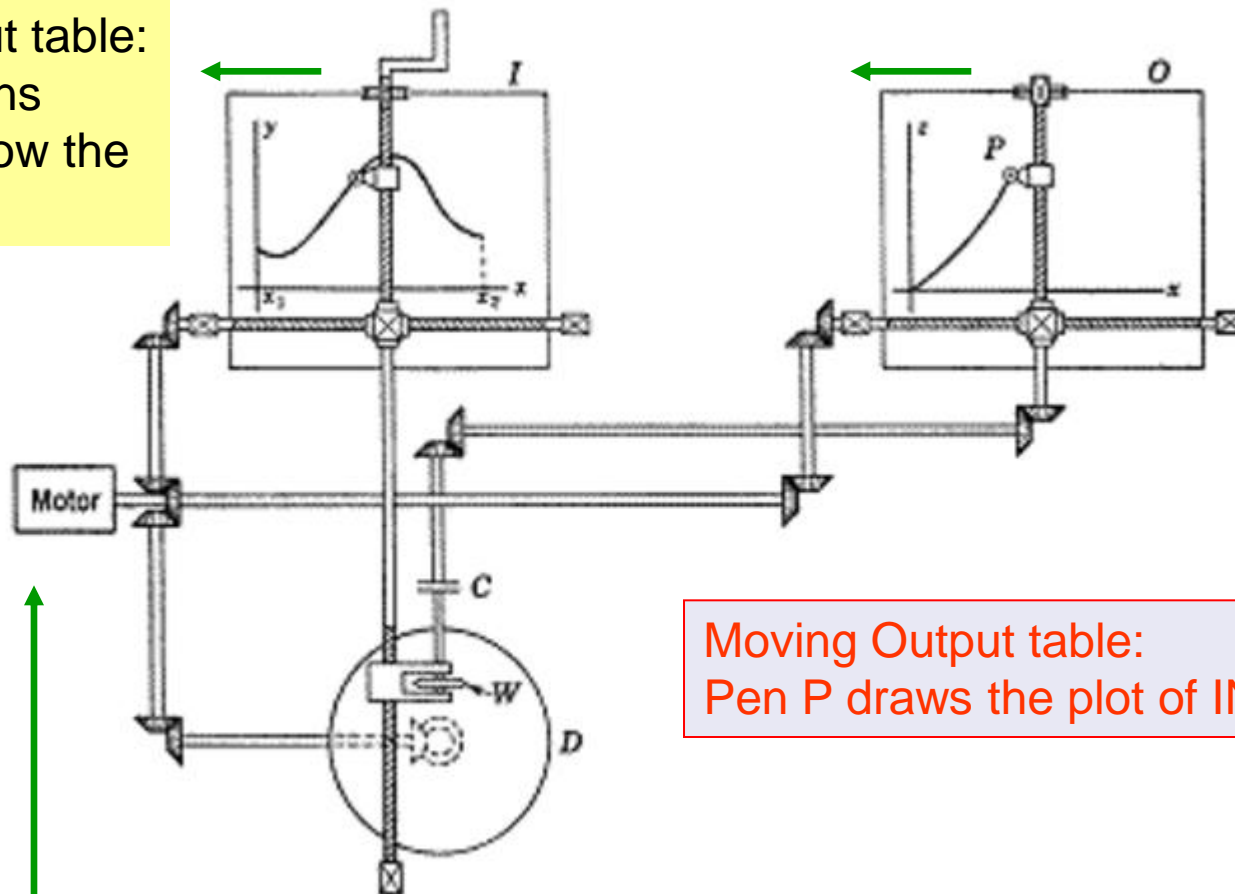




# Example of an integration



Moving Input table:  
operator turns  
crank to follow the  
curve  $f(t)$



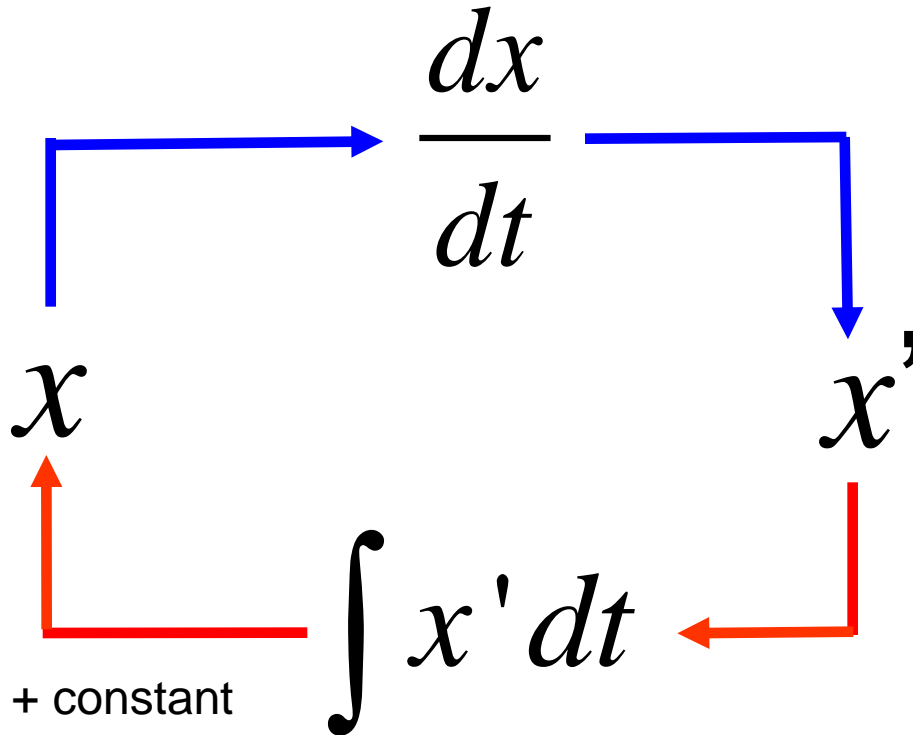
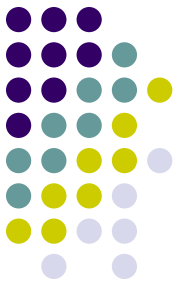
Moving Output table:  
Pen P draws the plot of  $\text{INT}[f(t)]$

Motor gives time  $t$ : spins integrator and makes  
progression on the horizontal axis of the tables



# Integration and differentiation

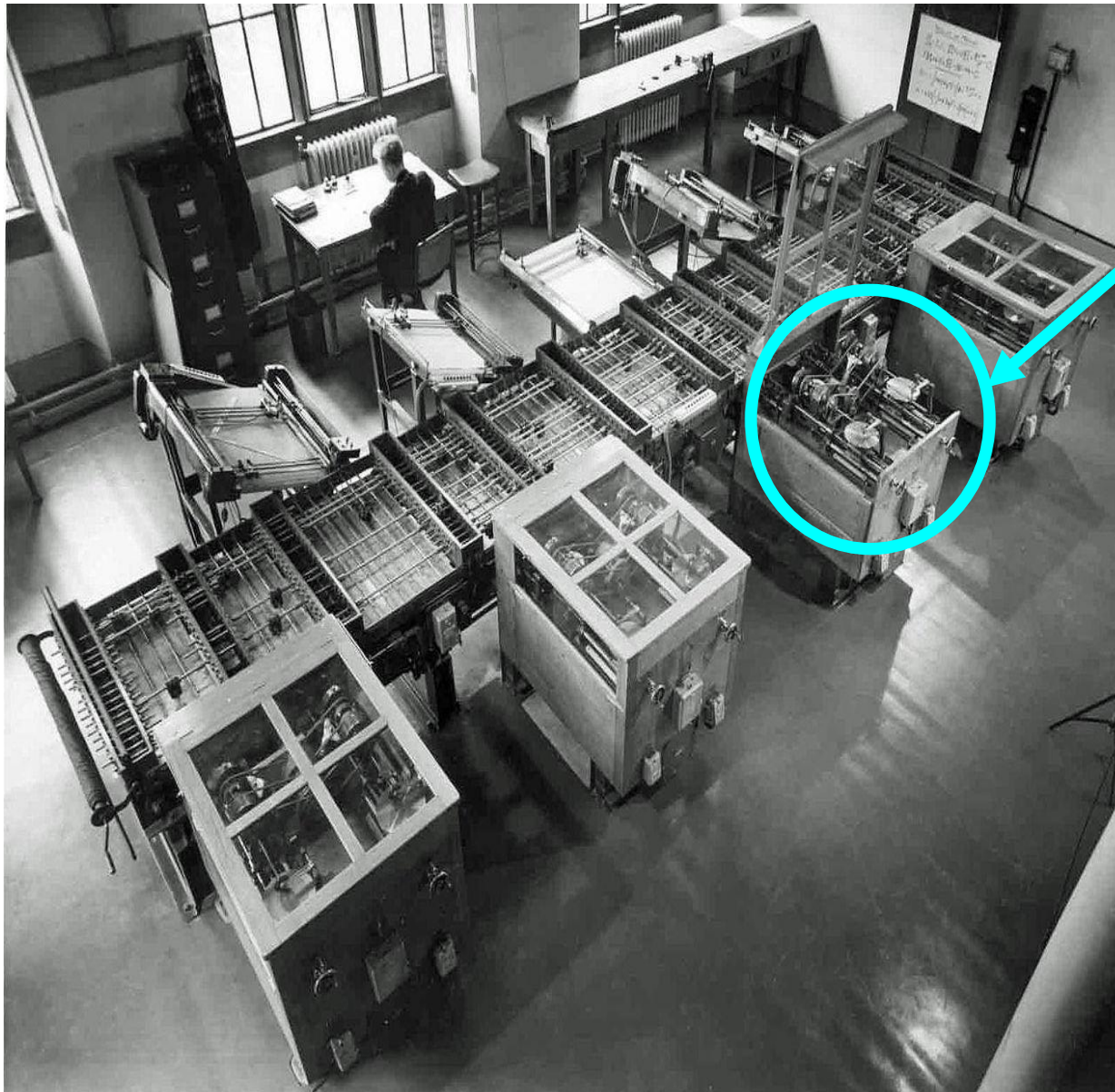
$x(t)$  is a function of time



**Differentiation** (derivation) and **integration** are inverse operations.

**Differential analysers** use integrators to solve differential equations, i.e. equations containing derivatives.

# Differential analyser (1)



disc integrators

Solving differential equations by integration:

1. Vannevar Bush, MIT, 1931
2. Rebuilt using Meccano elements by Douglas Hartree & Arthur Porter, Manchester Uni, 1934.

# Differential analyser (2)

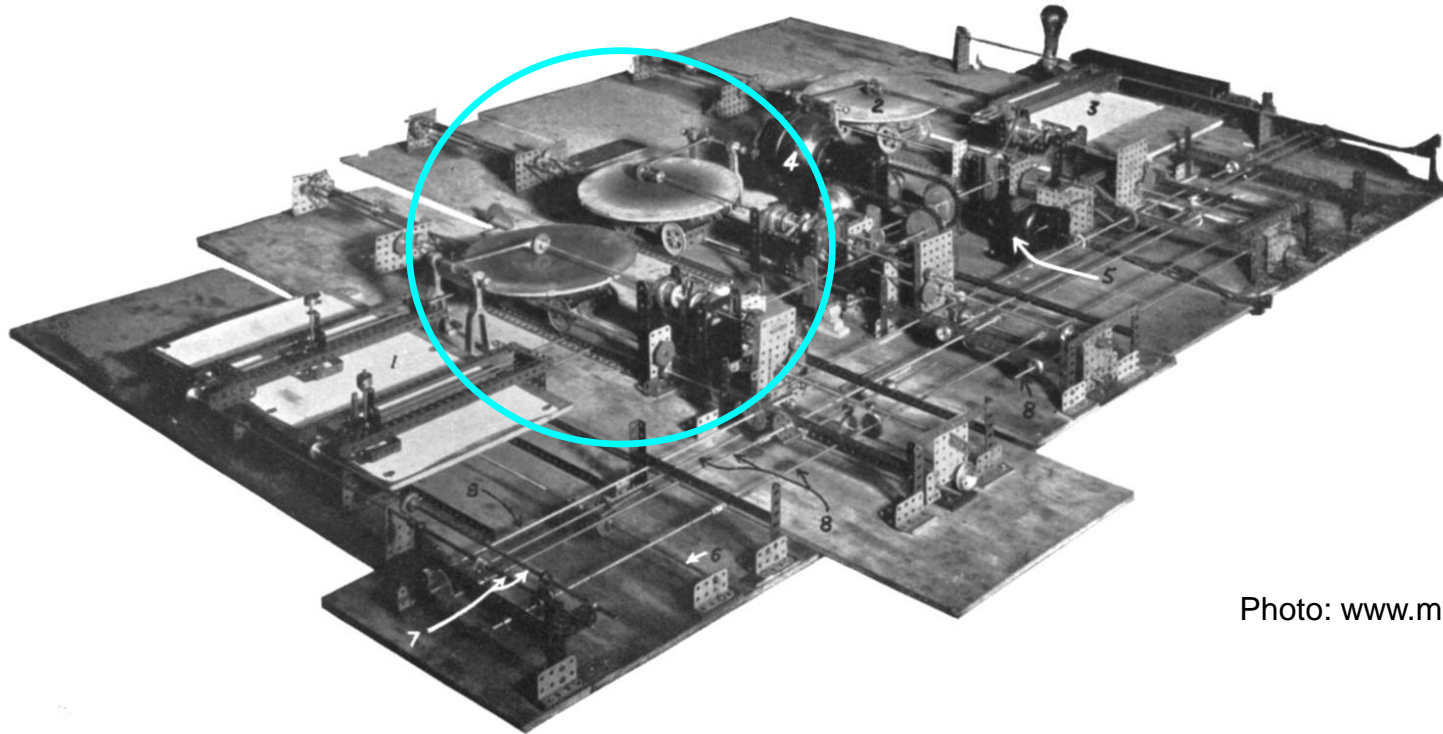
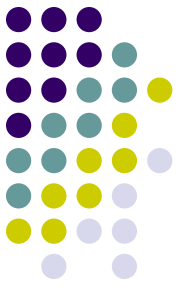


Photo: [www.meccano.us](http://www.meccano.us)

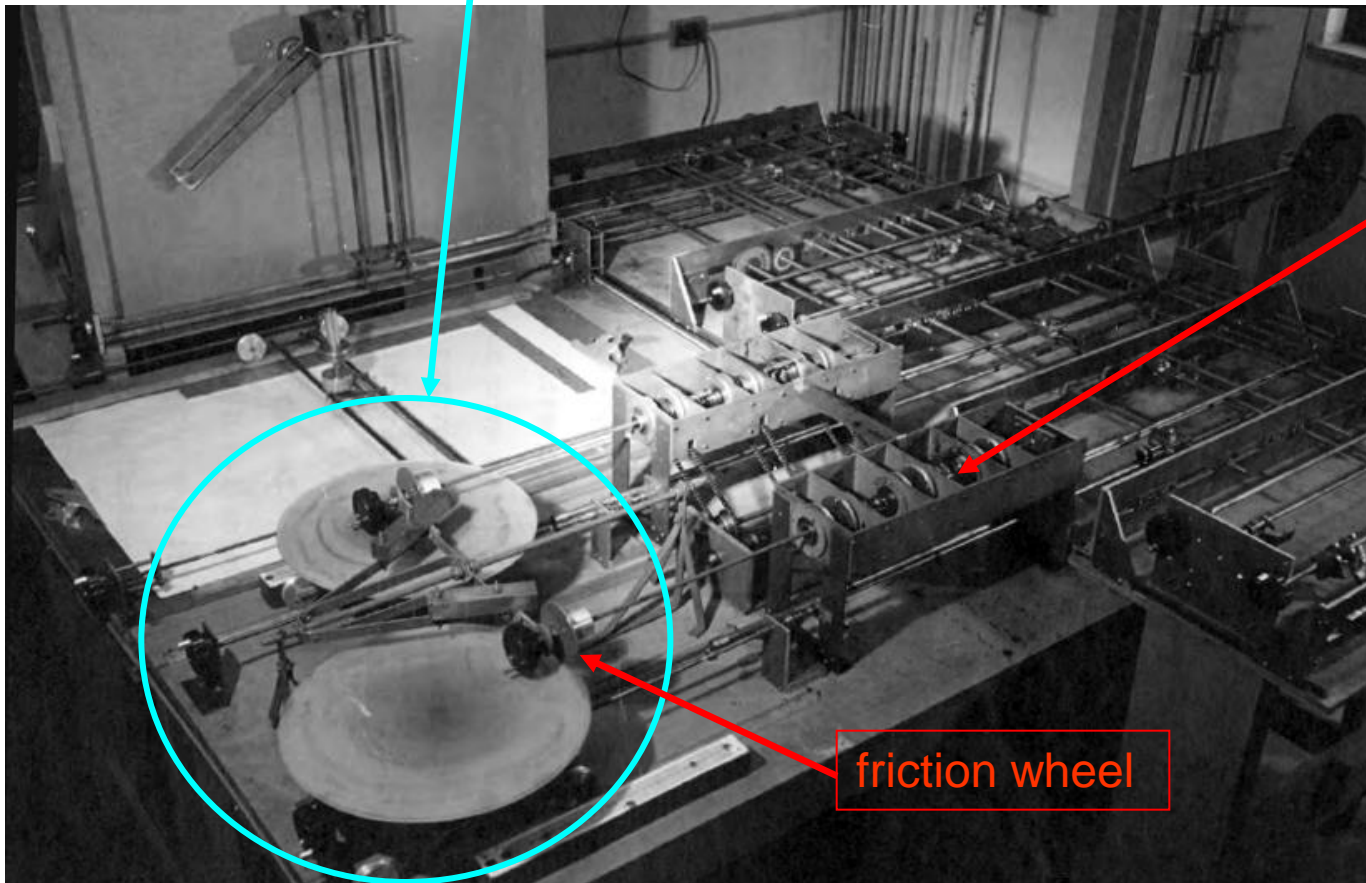
The 4<sup>th</sup> integrator module of the Hartree-Porter D-A.

The Meccano D-A was probably used by Dr. Barnes Wallis to solve the “bouncing bomb” problem.  
(WWII 1943, operation “Chastise” = bombing of Ruhr dams,)

# Differential analyser (3)



disc integrators (here the large discs move, the absolute position of the friction wheels does not change)



torque  
amplifier

friction wheel

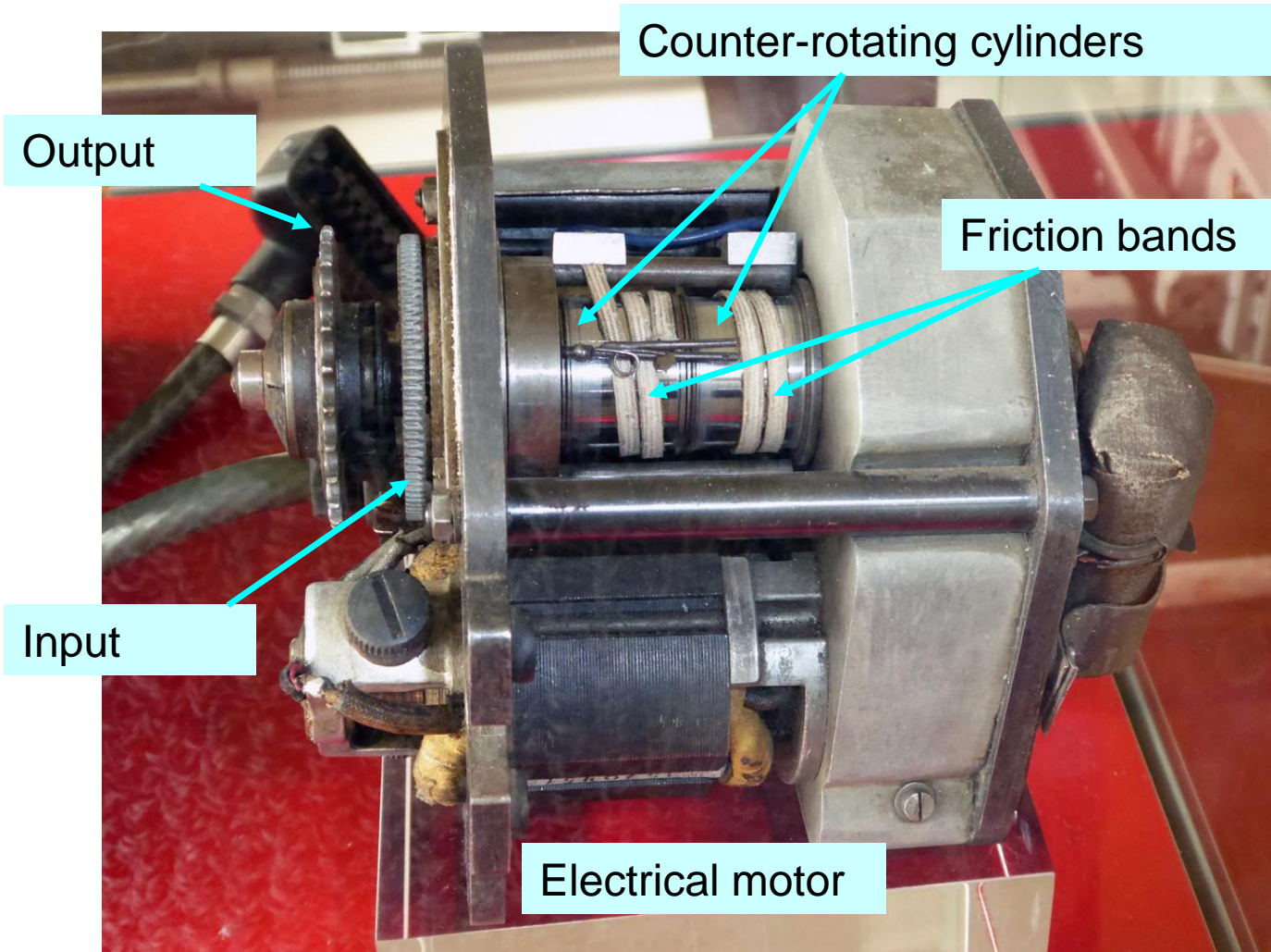
J.E.Kaspar  
thesis 1955

Iowa State  
University



# Differential analyser (4)

Torque amplifier (Drehmoment-Verstärker)



Invented by  
H.W.Nieman, 1925

Photo from  
Deutsches Museum

# Application: train graphs



Die ersten Integrieranlagen waren »Fahrdiagraphen« für die Eisenbahn.

Fahrgeschwindigkeit und Fahrzeit  
längs der Fahrstrecke  
in Abhängigkeit vom Streckenprofil,  
von der Lokomotiv-Zugkraft  
und vom gezogenen Gewicht  
waren in einem »Fahrdiagramm« aufzuzeichnen.

Knorr entwickelte 1914 den ersten Fahrdiagraphen.  
Er verwendet zwei Schneidenrad-Integraphen,  
deren Schneidenräder auf einem Zylinder abrollen.

Der Zweifach-Integrator zeichnet die Lösung  
der Differentialgleichung

$$y'' = f(y') + g(y) + h(x)$$

$f$ ,  $g$  und  $h$  sind gegebene Funktionen.

From  
Deutsches Museum

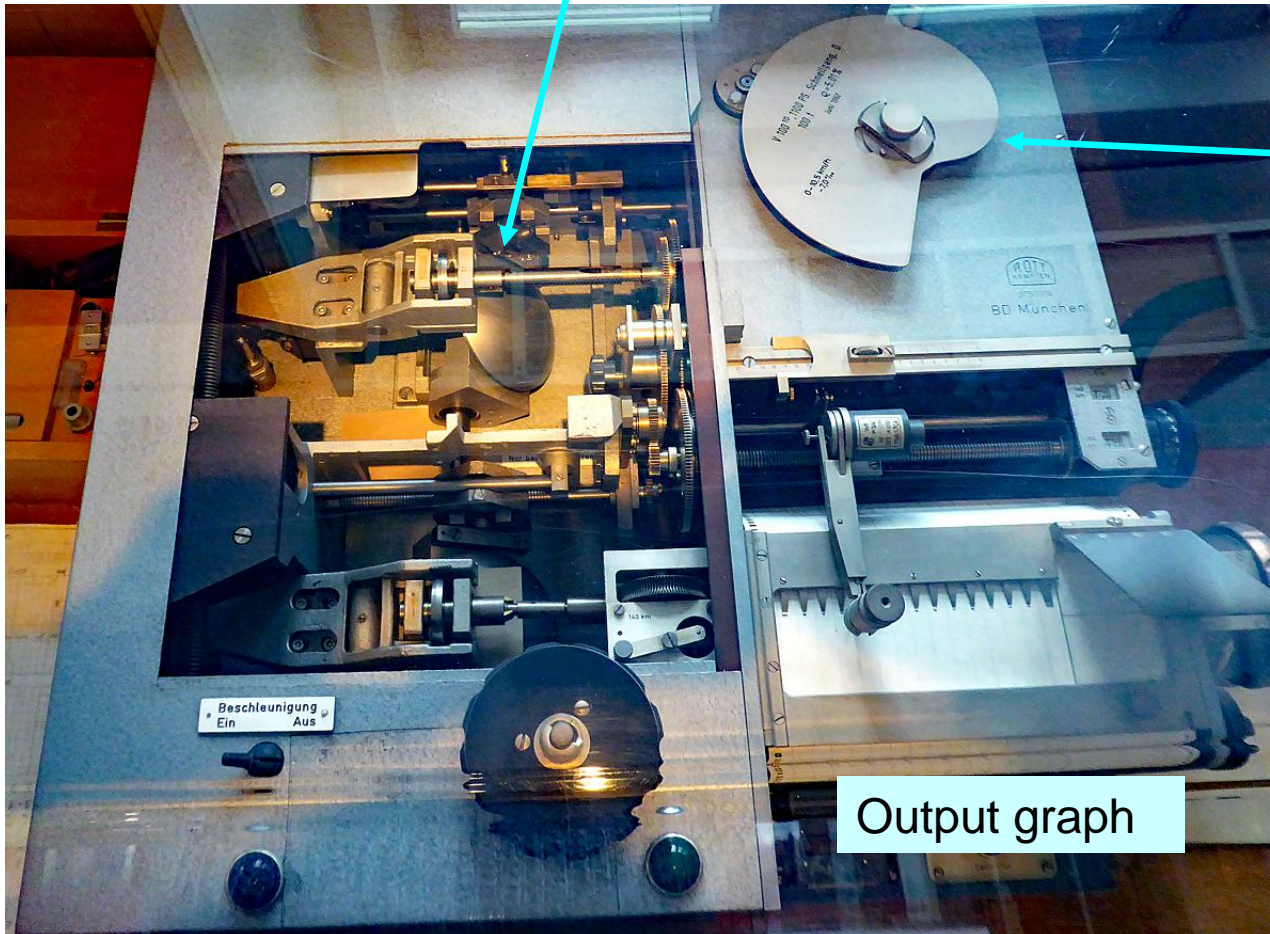


# Conzen-Ott “Fahrzeitrechner”



Spherical calotte integrator

Function-generator  $h(x)$

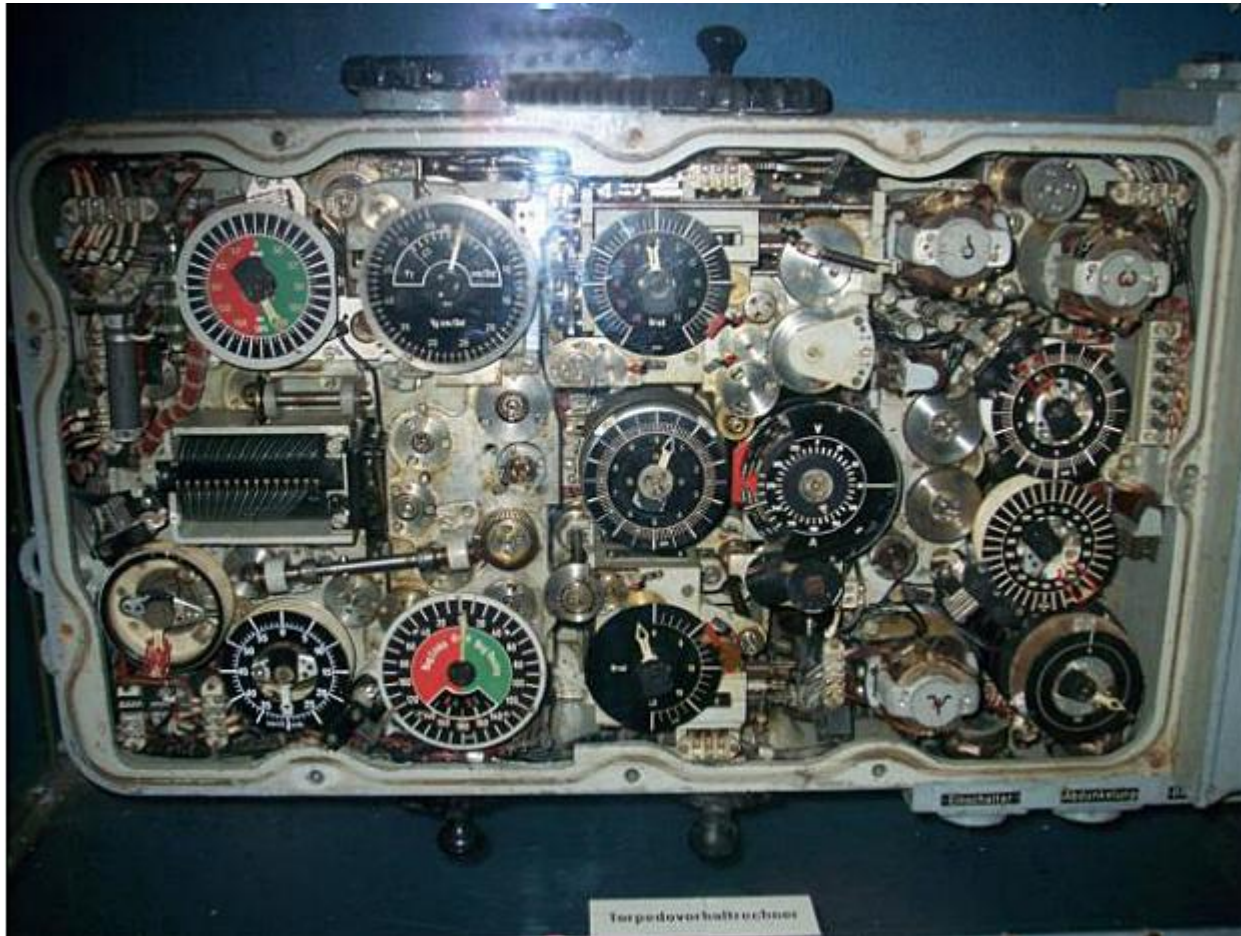


Output graph

Introduced in 1943.  
In use at the  
Deutsche Bahn  
up into the 1980's.

Photo from  
Deutsches Museum

# Torpedovorhaltrechner U-995



Thomas Müller:  
Analogrechner auf  
deutschen U-Booten  
des Zweiten  
Weltkrieges  
(Dissertation und  
Taschenbuch, 2015)

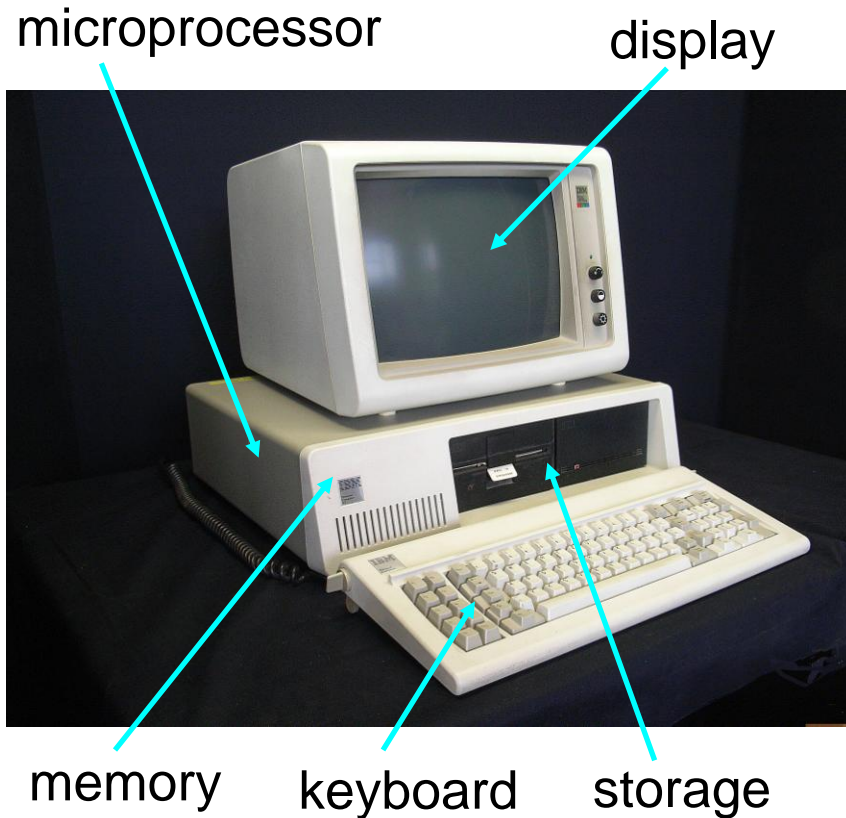


# Mark III Torpeda Data Computer

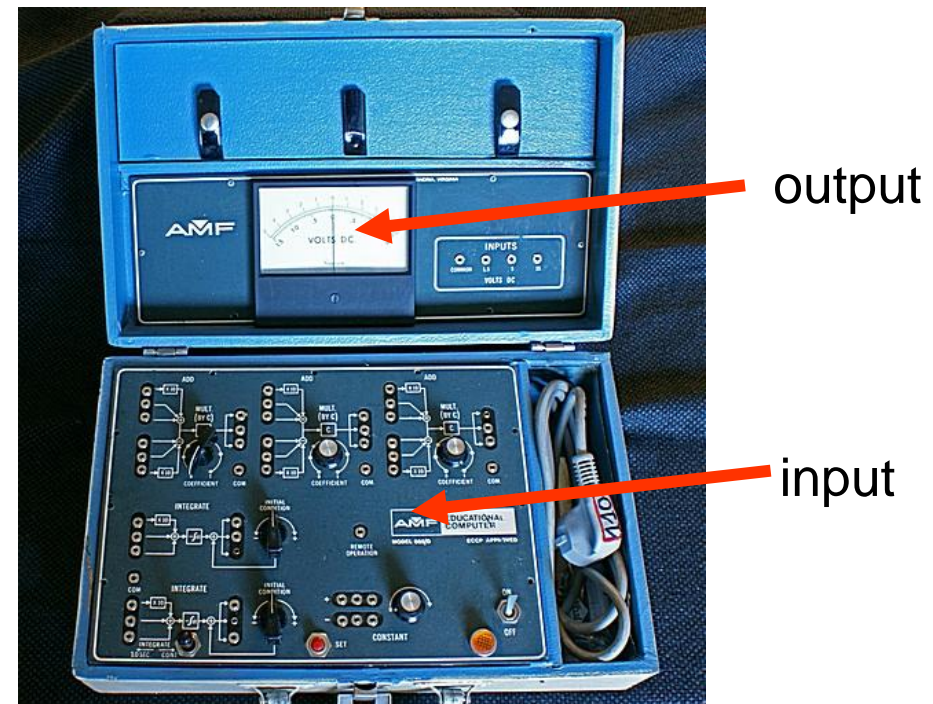


The Mark III was the standard analog computer for torpedo firing and guidance on US submarines in WWII  
[Google YouTube for "Torpedo Data Computer \(TDC\)"](#)

# Electronic analog computers

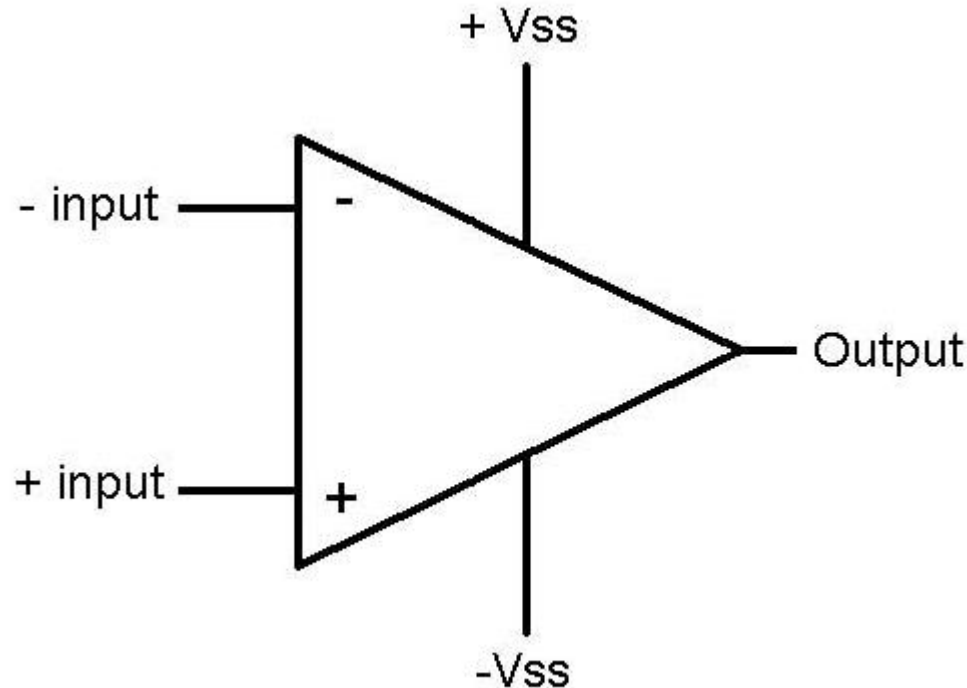


Digital computer (IBM XT, 1981)



**no** microprocessor, **no** display  
**no** memory, **no** keyboard,  
**no** storage  
Analog computer (AMF, 1970)

# One fundamental component: the Operation Amplifier (OA)



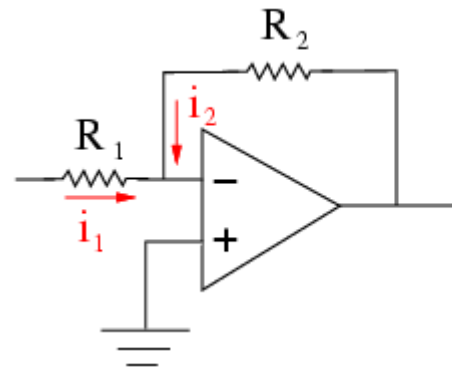
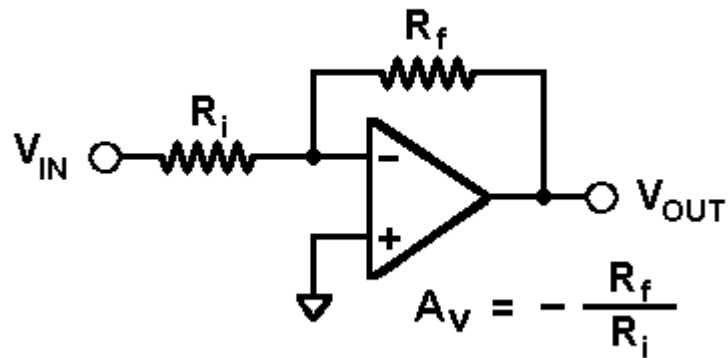
- **Voltage amplifier** with very high gain (typ. 100000 to 1000000)
- usually  $-$ input is used, output voltage is inverted
- **input current virtually 0**
- two supply voltages  $V_{ss}$ , typical +15 and -15 VDC



# OA is an inverting amplifier



Inverting Amplifier



$$i_1 + i_2 = 0$$

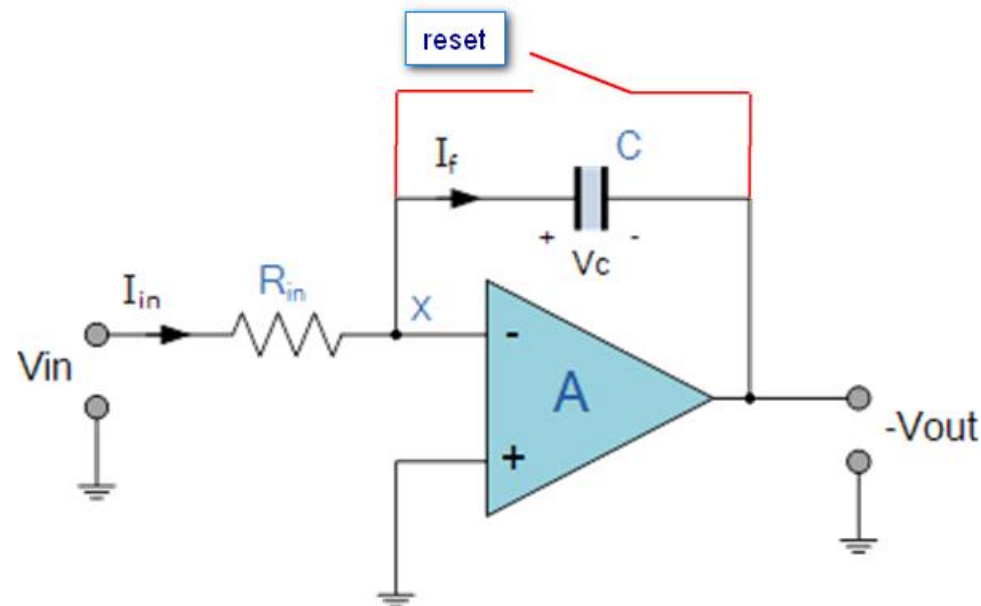
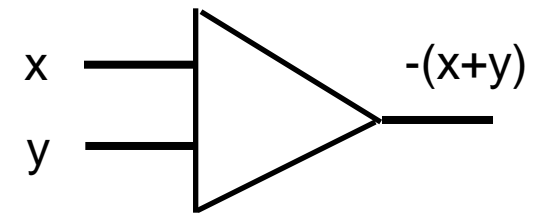
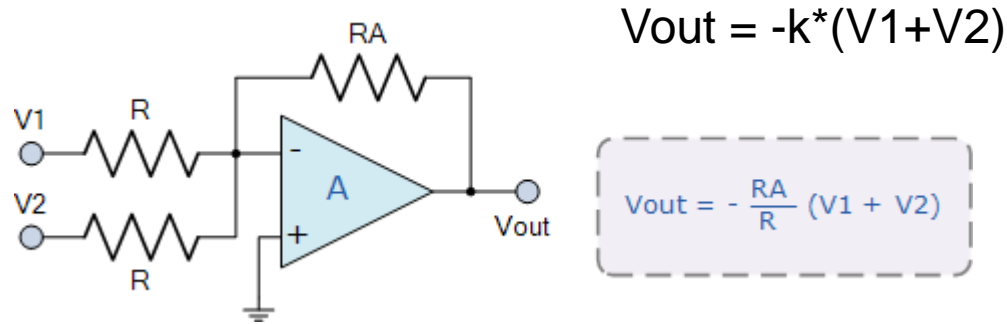
$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

$$V_{out} = -k \cdot V_{in}$$

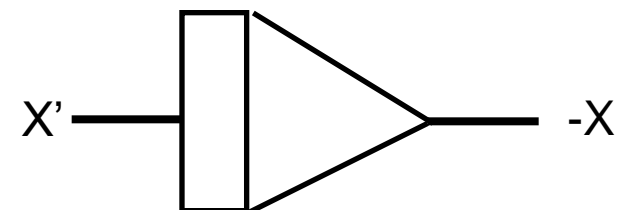
Current  $i_i \sim 0$  mA

# OA circuits: adder, integrator

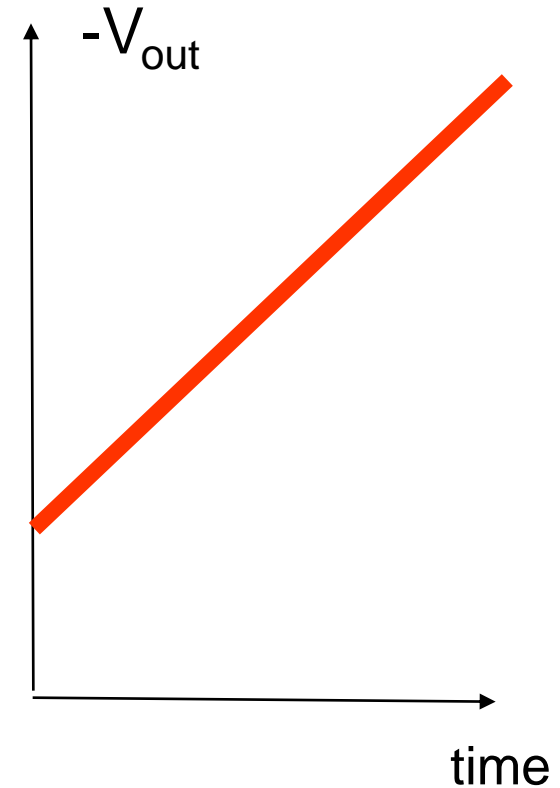
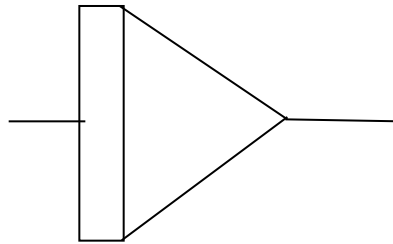
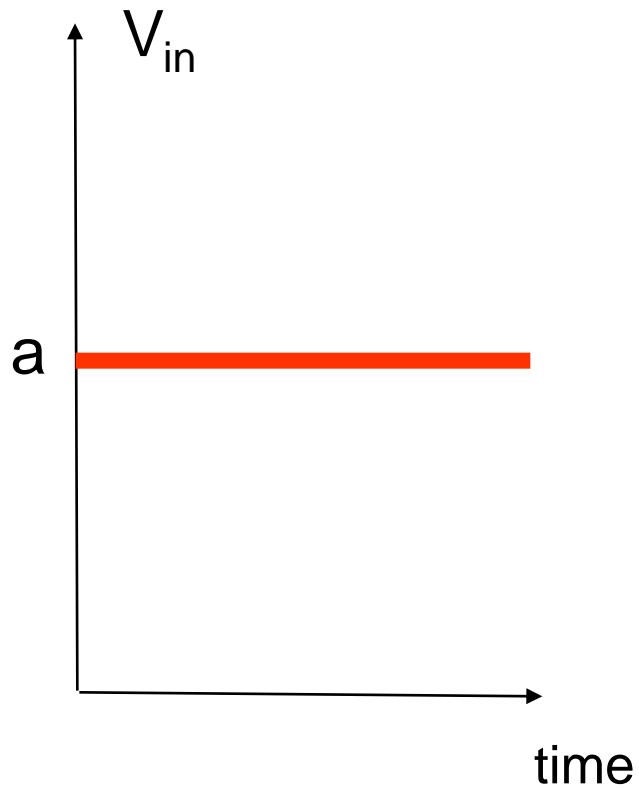


$$V_{out} = -k * \int_0^t V_{in} * dt$$

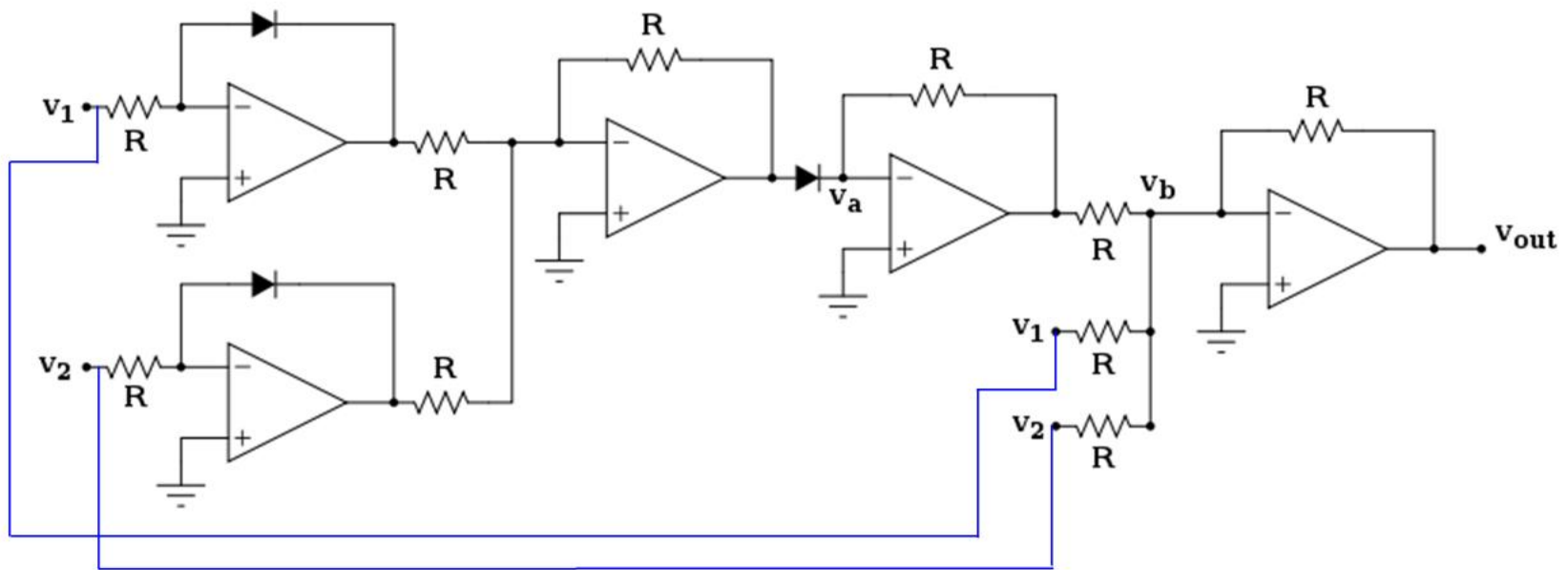
$$k = 1 / (R_{in} * C)$$



# Example of integration



# OA circuits: multiplier



$$V_{out} = -k^*(V_1 * V_2)$$

# History of OA's (1)

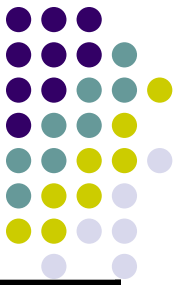
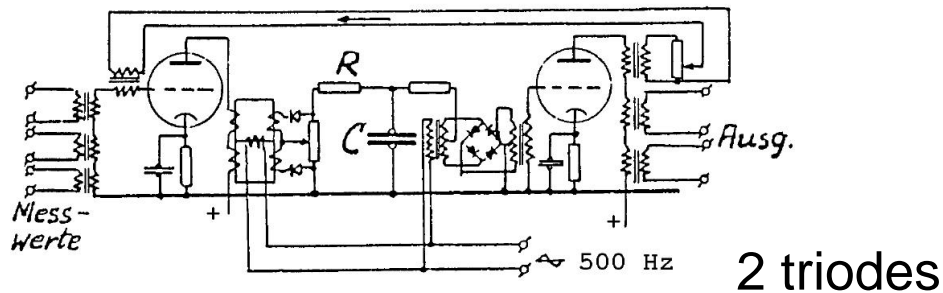


Abb. 1

*„Integrator“ mit Rück-  
kopplung und Modu-  
latorverstärker*



Helmut Hoelzer,  
Peenemünde, 1941  
Germany



1 dual triode  
1 pentode/triode

K2-GW  
first commercial OA  
G.A. Philbrick, 1952  
USA

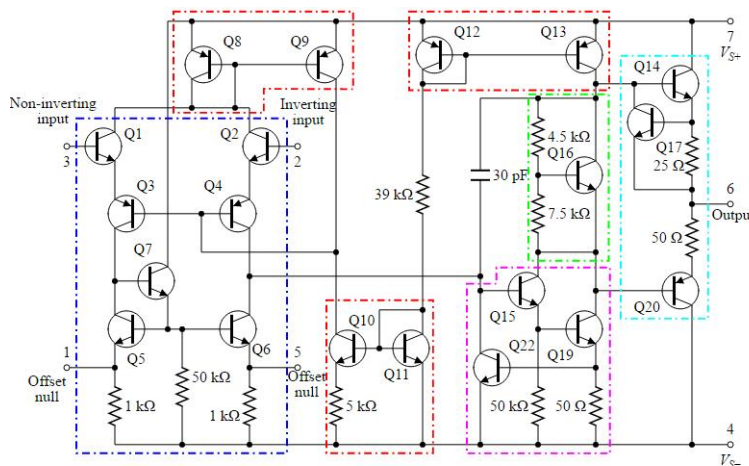


# History of OA's (2)

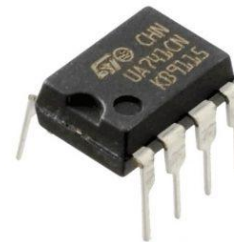


uA702  
Fairchild  
Semiconductors  
1964, USA  
first IC Opamp

uA709  
Fairchild Semiconductors  
1965, USA  
uA741  
Fairchild Semiconductors  
1965, USA



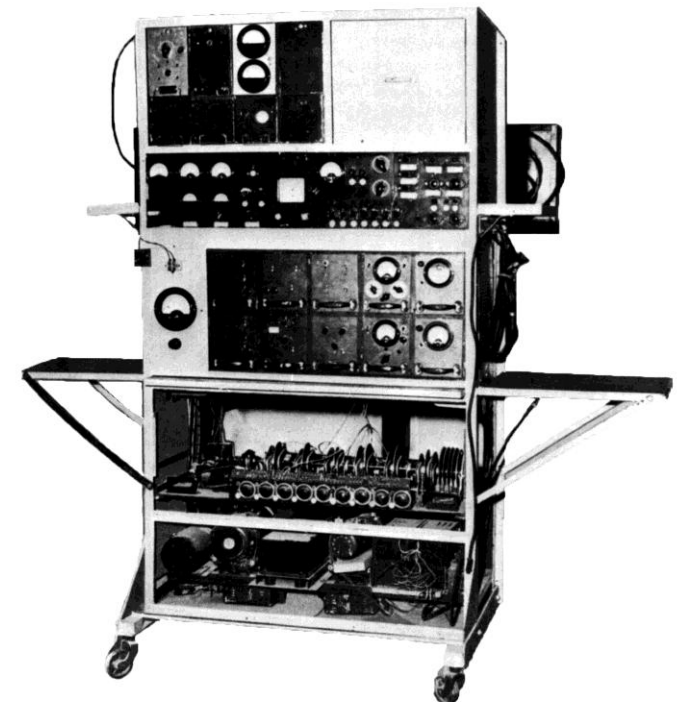
uA741 most successful OA  
of all times!



# Helmut Hoelzer (1912-1996)



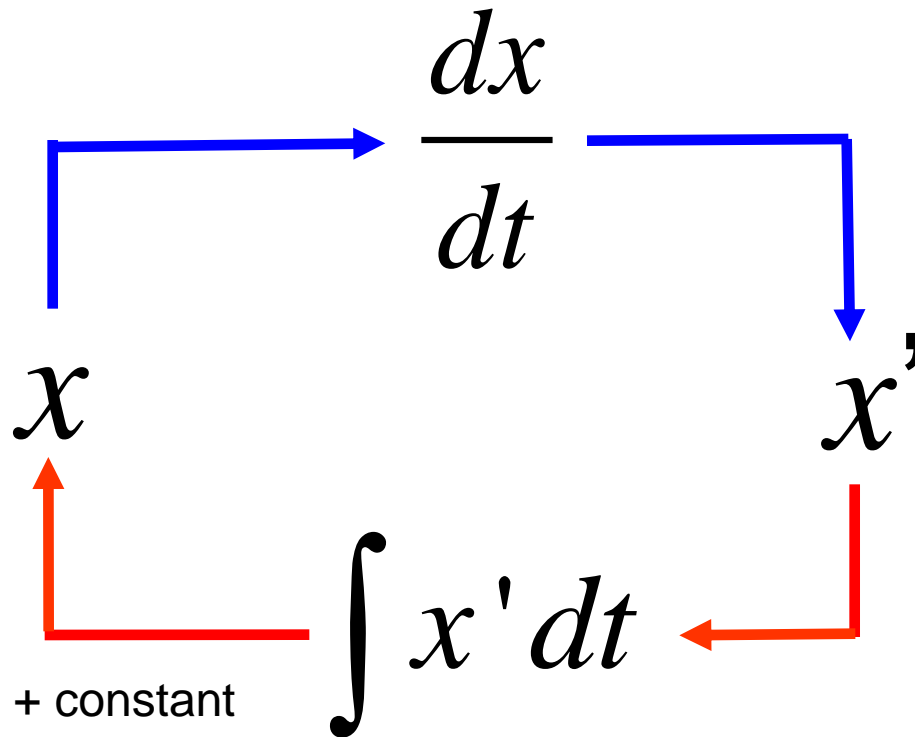
- Works at Peenemünde with Wernher von Braun on the A-4 rocket (V2 = “Vergeltungswaffe” 2)
- Invented and built in 1941 the first electronic analog computer
- Invented and built the “Mischgerät” for guiding the V2
- After WWII emigrated to the USA; worked on rockets and related mathematics (Marshall Space Flight Center).



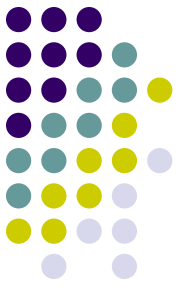


# Remember:

Differentiation (derivation) and integration are inverse operations



# One of the V2 problems

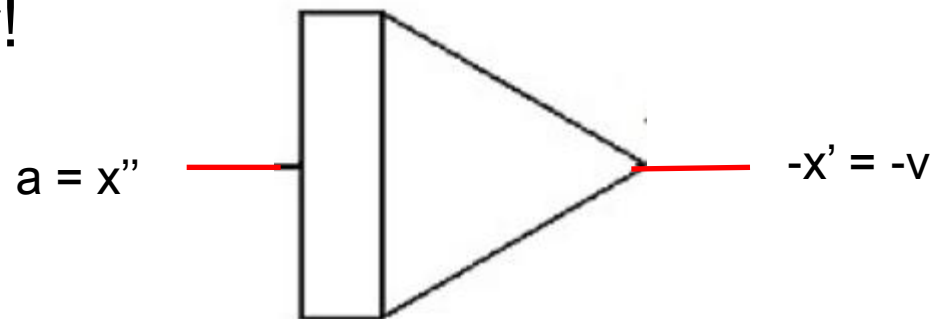


- How to measure the absolute speed?
- Hoelzer's idea: acceleration is easy to measure; speed is the integral of acceleration, so invent an integrator!

$$v = \frac{dx}{dt} = x'$$

$$a = \frac{d^2x}{dt^2} = \frac{dx'}{dt} = \frac{dv}{dt}$$

$$v = \int a * dt$$



# Another V2 problem: steering



Jet spoilers (“Strahlruder”, graphite pads) used during lift-off when speed is low or later outside the atmosphere.



Fin spoilers (“Flügelruder”) used at higher speeds in lower atmosphere.



# V2: gyro and Mischgerät



One of the two gyroscopes of the V2 (pitch and roll control for lateral stabilization)

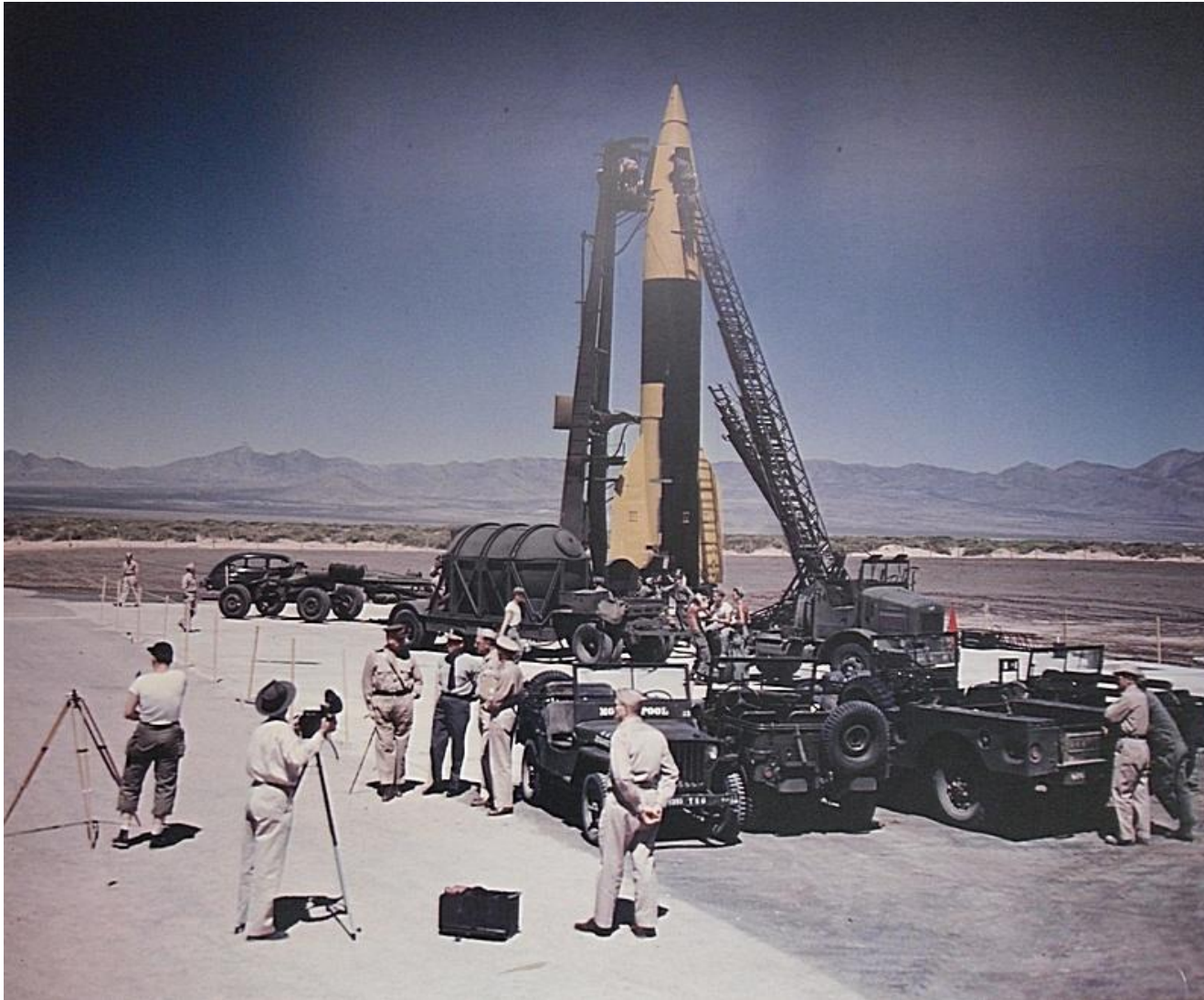
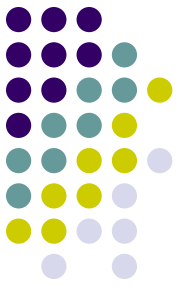


Hoelzer's "Mischgerät" = **analog guidance computer** of the V2; located in the head of the rocket. It takes signals from the gyroscopes and acceleration sensors, commands the two types of spoilers and stops the engine at speed  $v$ .



V2 at White Sands Proving Grounds Museum (New Mexico). The rocket has a length of ~14m. The engine runs for about 1 minute (ethanol and oxygen), the top of the trajectory is ~90 km.

# V2 launch at White Sands



Launch at White Sands of one of the many V2 taken back to the USA. Probably ~1946.

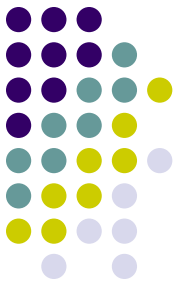
# Analog (electronic) computer



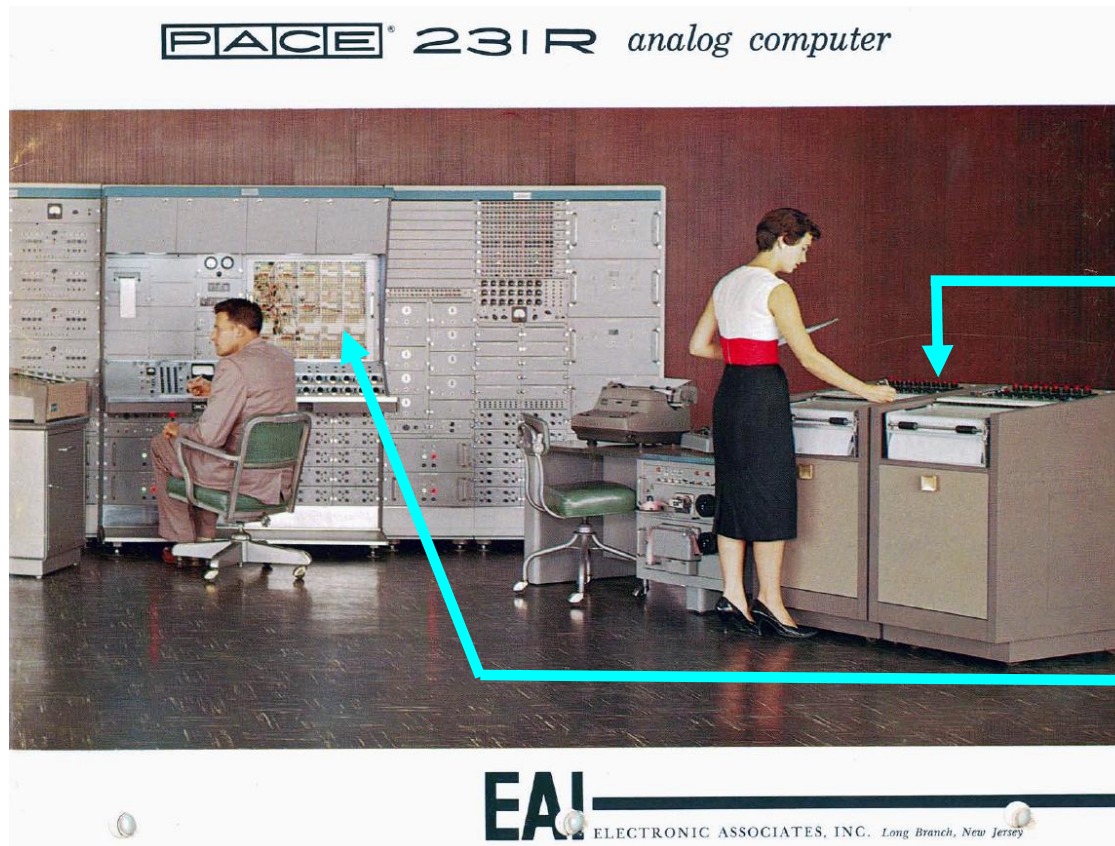
- Uses a **model** of the process to be solved. Variable structure, depending on program!
- Application: process control and simulation: chemical and nuclear reactors, flight, fluid flow, epidemics...
- Mostly used from 1950 to about 1970's
- Works in real time and in parallel, up to 100000 times faster than the first digital computers



# Some examples of analog computer manufacturers



EAI (Electronic Associates Incorporated, New Jersey, USA, \*1945)  
European headquarter in Brussels. First analog computer in 1952.



**PACE** 231R analog computer

EAI (Pace) 231R  
Computer, 1961

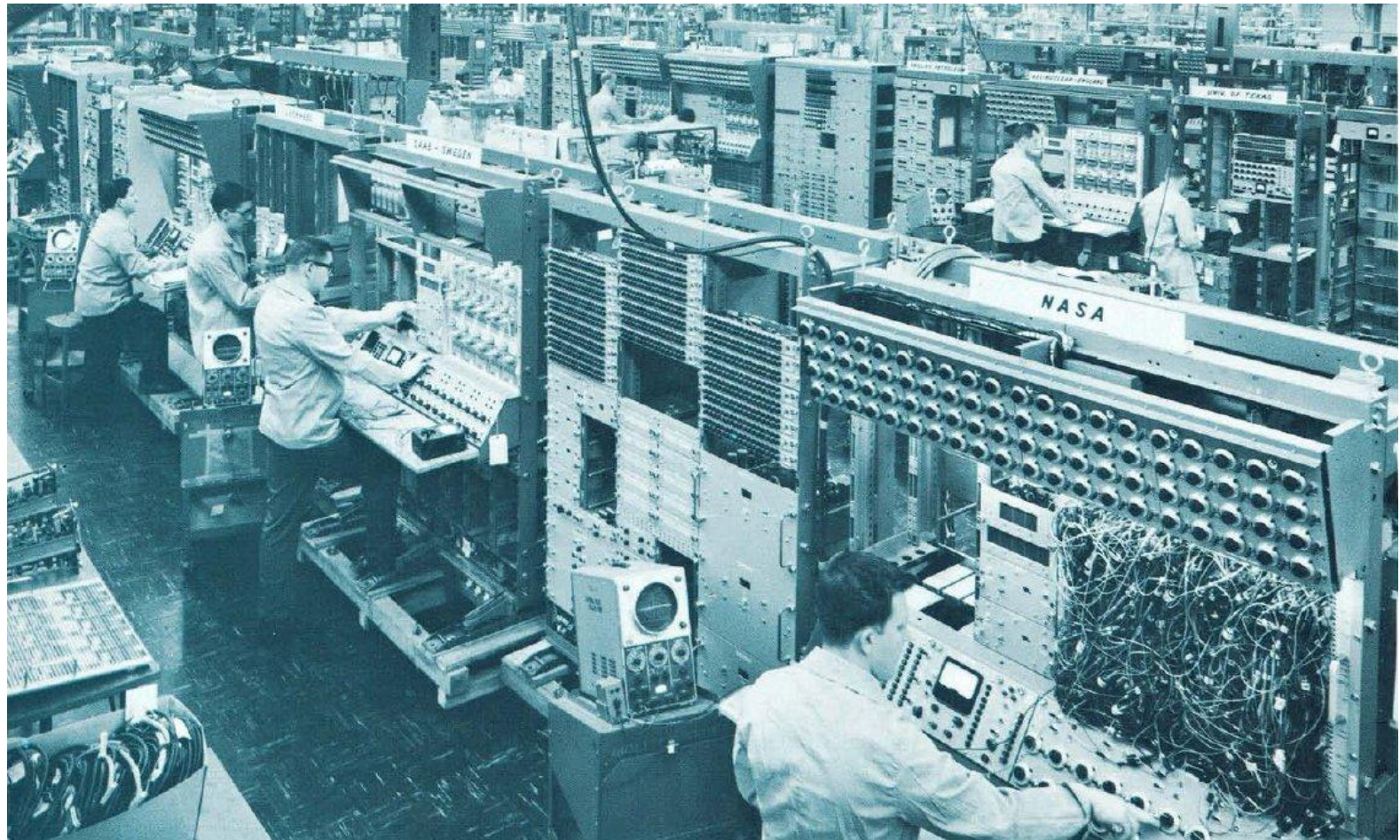
Plotter for output

Pierre DAVID, a  
former LCD  
student, worked  
at EAI, Brussels.

Patch panel for  
programming



# EAI analog computers (2)



Assembly line



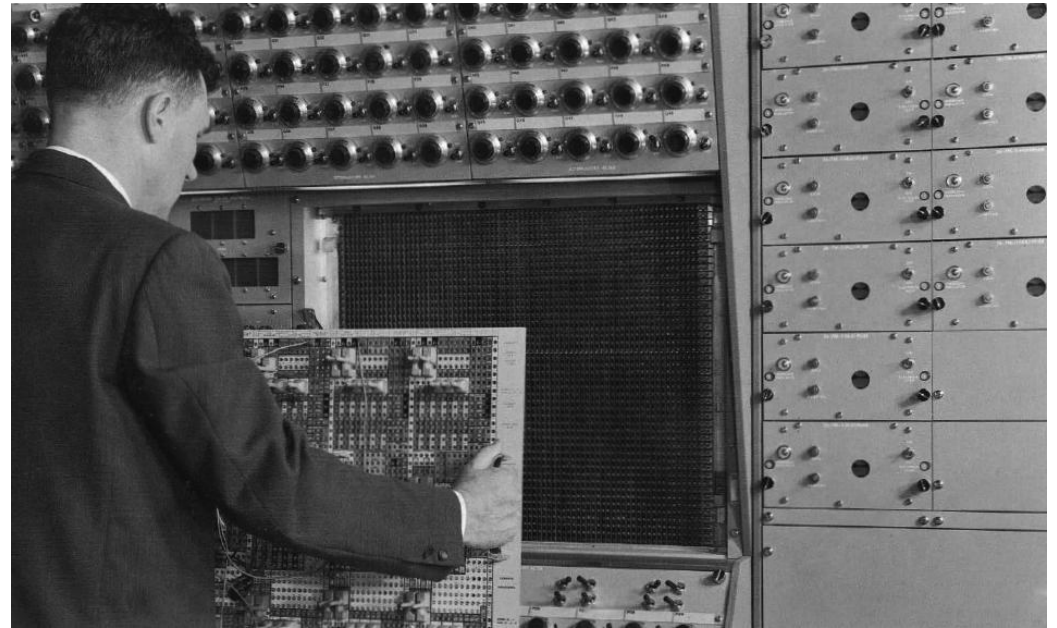
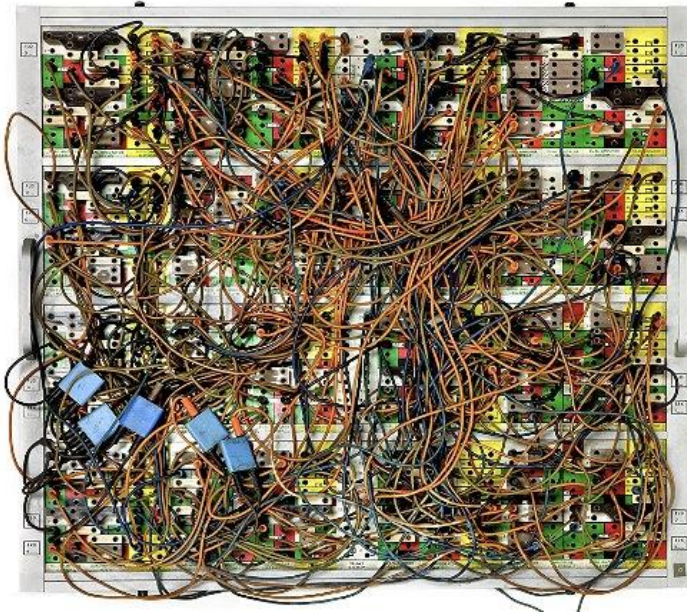
# EAI analog computers (3)



EAI (Pace) TR-48  
desktop computer  
1962

The **potentiometers** are used to define the various parameters of the model. Correctly calculating the settings was one of the big difficulties of the analog computers.

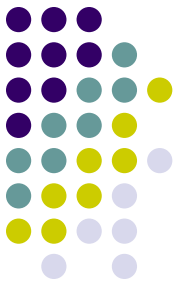
# EAI analog computers (4)



The spaghetti wiring of a patch panel. Bigger computers often had removable panels for storing the wired programs (right: EAI Pace 231R)



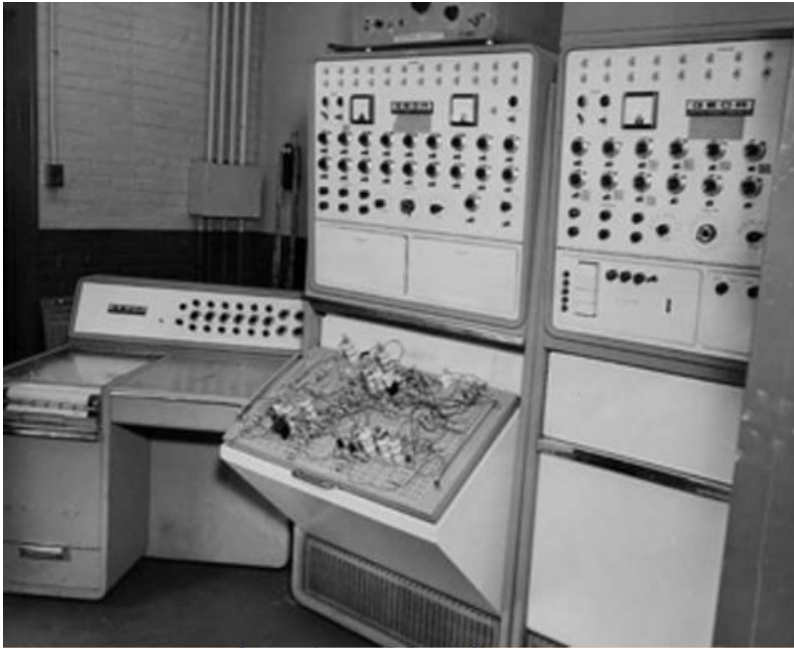
# EAI analog computers (5)



EAI HIDAC2400  
Hybrid computer, 1963.

Analog hybrid computers were a mix of both worlds. The digital part allowed for instance to calculate the setting of the potentiometers and often to set them automatically by servomotors.

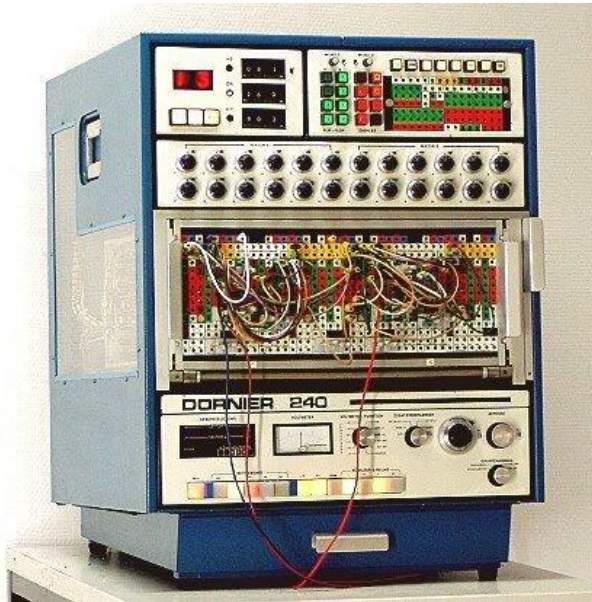
# Goodyear analog computers



Goodyear Aerospace Corp. developed a range of analog computers called GEDA = **Goodyear Electronic Differential Analyzer**

This model is from 1953.

# Dornier analog computers



Dornier DO-240  
analog computer (~1970)

([www.technikum29.de](http://www.technikum29.de))

The aircraft constructor DORNIER (DE) started building analog computers to solve the problems related to VTOL planes (DO-31 E3, first flight 1967)





# Telefunken analog computers (1)



Telefunken RA-1  
First analog computer built  
by Telefunken in 1955.

(photo Prof. Bernd Ulmann)

Two oscilloscopes to  
visualize the results

# Telefunken analog computers (2)



Telefunken RA-770

A very precise analog computer (precision  $10^{-4}$ , weight 550 kg).

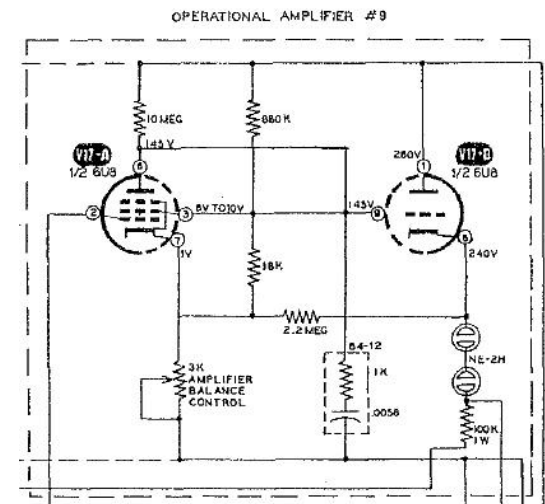
Used at the Forschungszentrum Jülich for nuclear research.

(photo Prof. Bernd Ulmann)



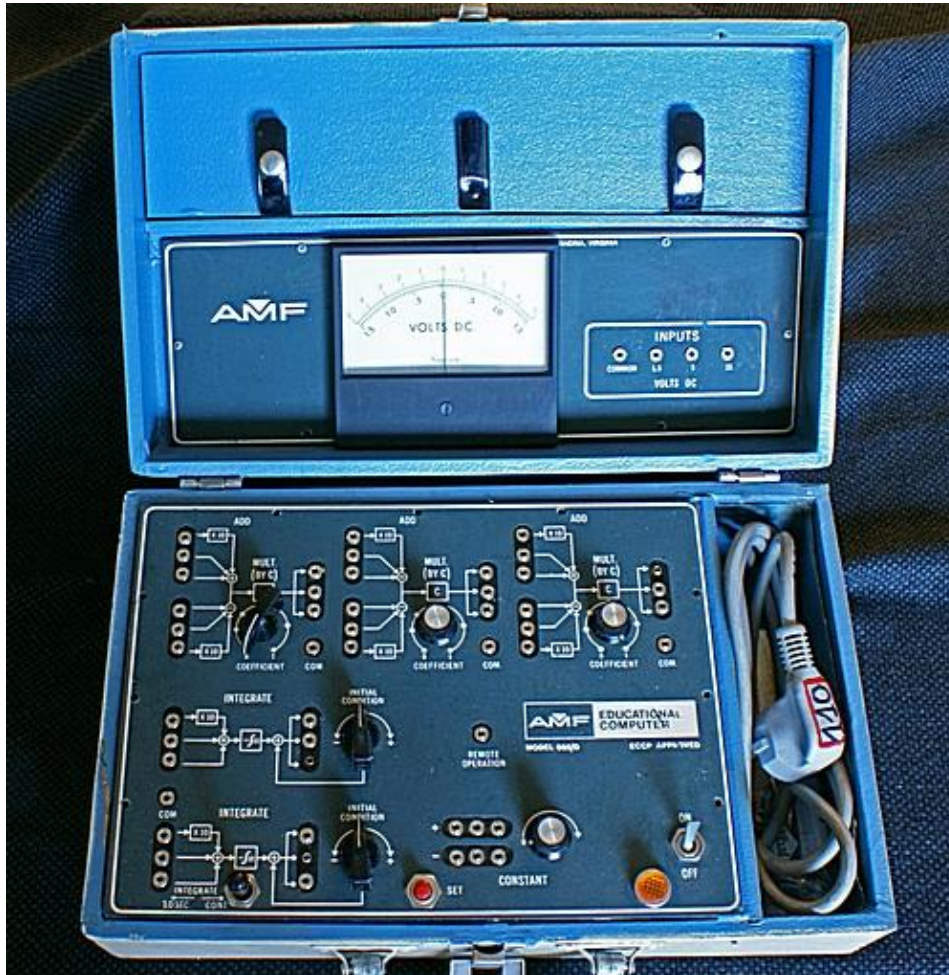


Heathkit EC-1  
Educational computer  
1961, 9 OA with tubes



The R,C components to define the function (adder, integrator,...) must be added on the front-plane. Precision and stability are modest.

# analog computers (2)



AMF

(American Machine and Foundry: bowling, bicycles, tennis rackets, nuclear reactors for research...)

AMF 665/D educational computer from 1970.

OA = u741

2 integrators, 3 adders

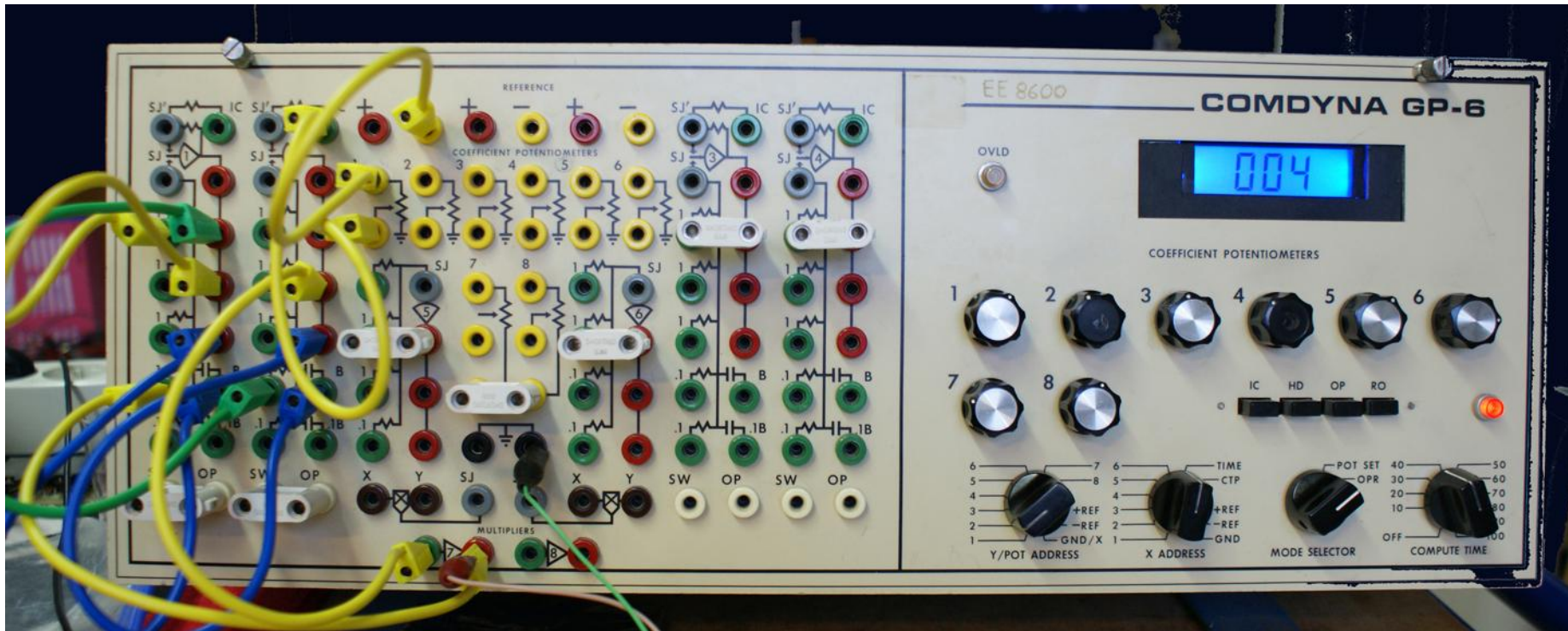
(donated by AALCD).





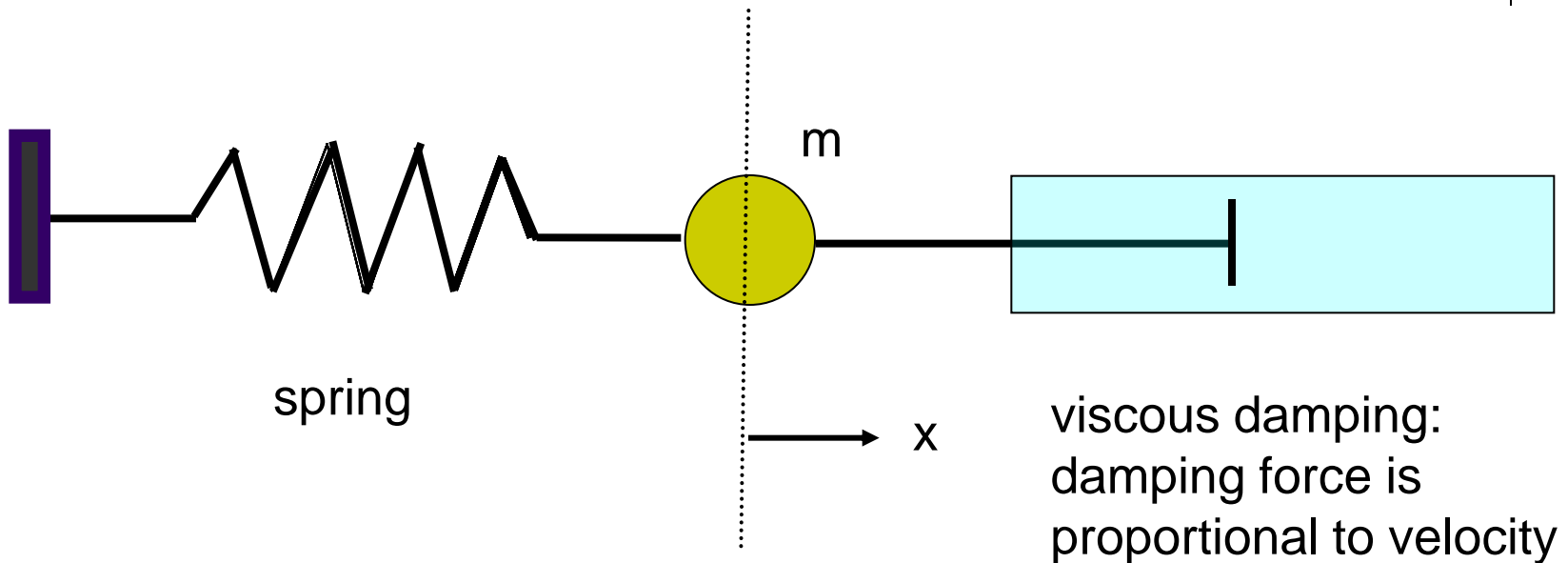
## COMDYNA GP-6

6 integrators, 2 multipliers, 2 inverters. Built from 1968 to 2004. OA = u741.  
Comdyna founder is Ray Spiess, a former EAI engineer.  
This specimen comes from the University of Wisconsin (donated by AALCD).



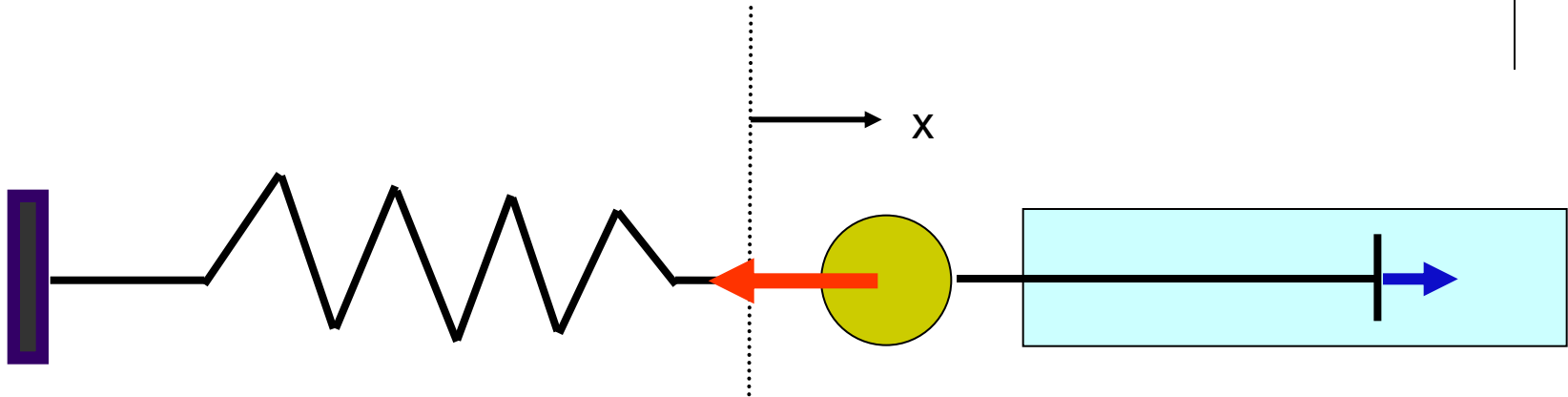


# Solving the damped oscillator problem (1)



To solve this problem we need a model!

# Solving the damped oscillator problem (2)



Spring force =  $-k*x$

damping force =  $-d*v = -d*x'$

Total force =  $-k*x - d*x'$

Newton: Total force =  $m*a = m*x''$   $\longrightarrow$   $m*x'' = -d*x' - k*x$

For simplicity:  $m = d = k = 1$ :

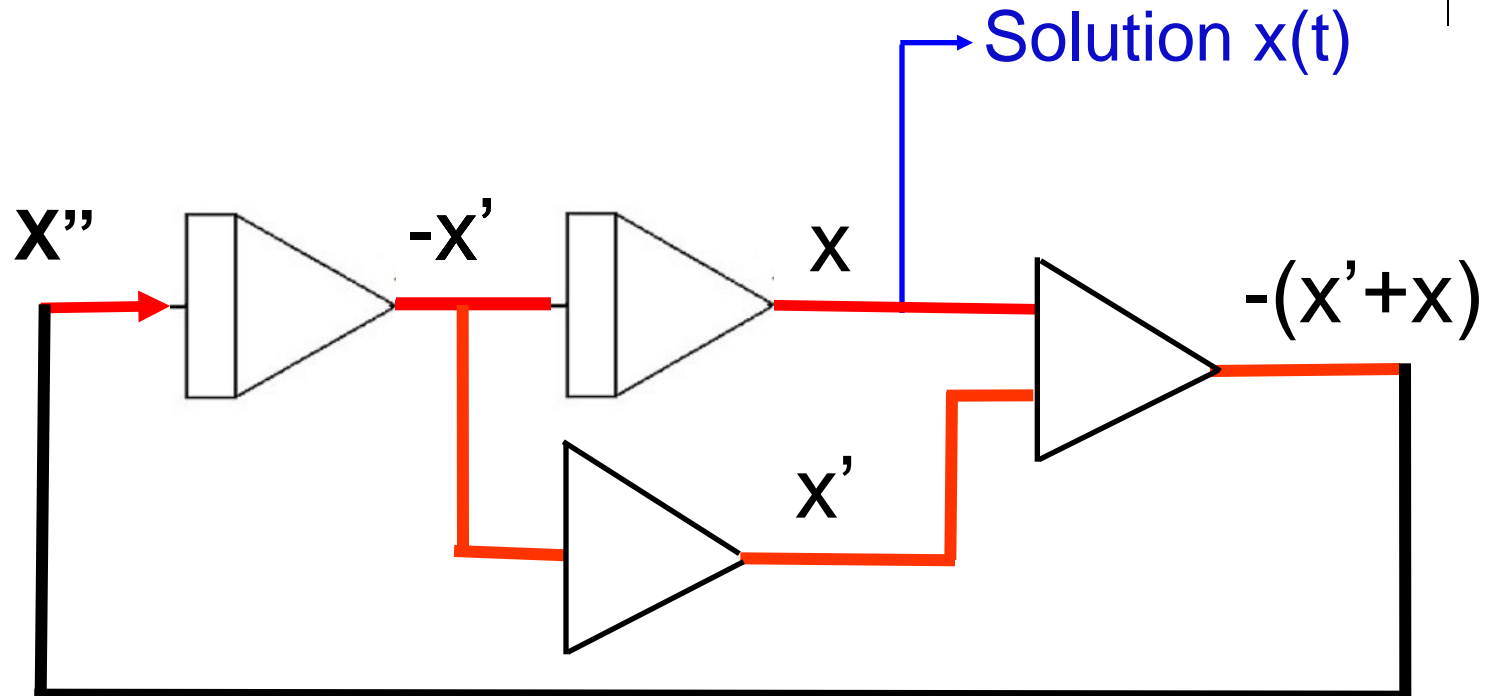
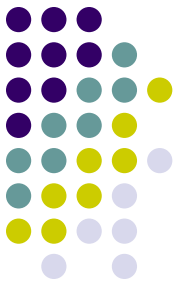
$$x'' = -x' - x$$

this is the model!

$x(t)$  is the solution to find...

# Wiring for analog computer

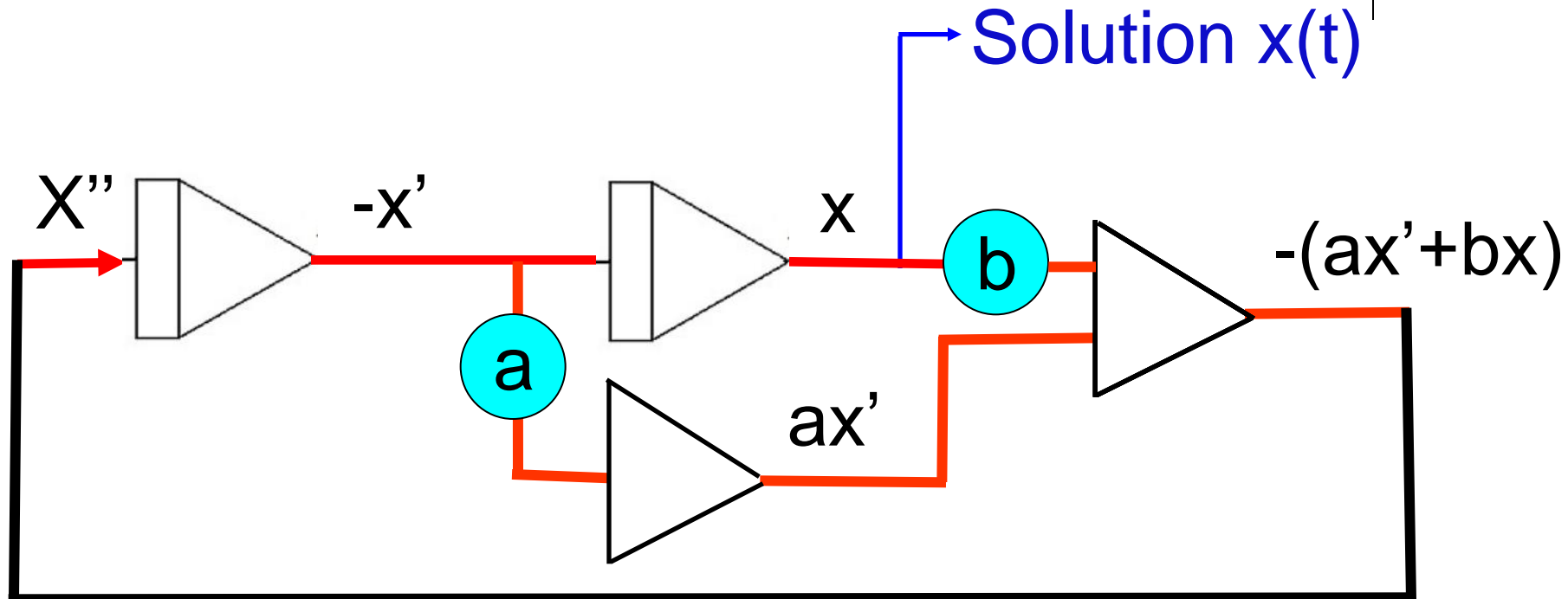
$$x'' = -x' - x = -(x' + x)$$



The feedback of the output to the input was first suggested by Lord Kelvin (William Thomson) in 1876

# Wiring for analog computer

$$x'' = -ax' - bx = -(ax' + bx)$$



**a**

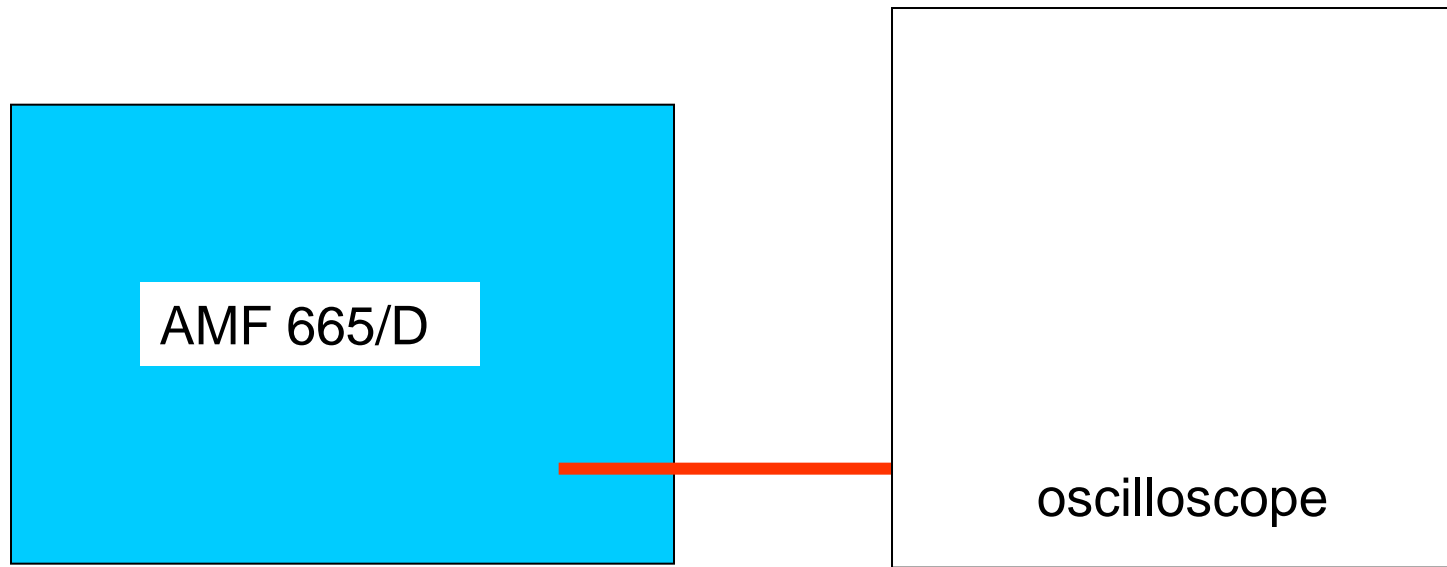
Pontentiometer related to viscous damping

**b**

Pontentiometer related to stiffness of spring

**The potentiometer can be changed during the running "program" !**

# Demonstration with AMF analog computer





# An “infectious problem” (1)



- Town has population of 1000
- Initially:
  - 10 are sick ( $y$ )
  - 900 may be become sick ( $x$ )
  - 90 are immune ( $z$ )
- Contact rate between sick and not yet sick people =  $1/1000$  (per day)
- $1/14$  of the sick become immune every day
- How do  $x$ ,  $y$ ,  $z$  evolve in time?

# An “infectious problem” (2)



- Contact rate between sick and not yet sick people = 1/1000 (per day):

$$x' = - 1/1000*(x*y) \quad [ \text{change per day of not yet sick people} ]$$



new infections per day

# An “infectious problem” (3)



- 1/14 of the sick people become immune every day:

$$y' = + 1/1000*(x*y) - 1/14*y \quad [ \text{change of sick people per day} ]$$



new infections per day



new immunizations per day

# An “infectious problem” (4)



- $z' = 1/14 * y$

[ change of people having recovered per day, now immune ]

Model:

$$x' = - 1/1000*(x*y)$$

$$y' = + 1/1000*(x*y) - 1/14*y$$

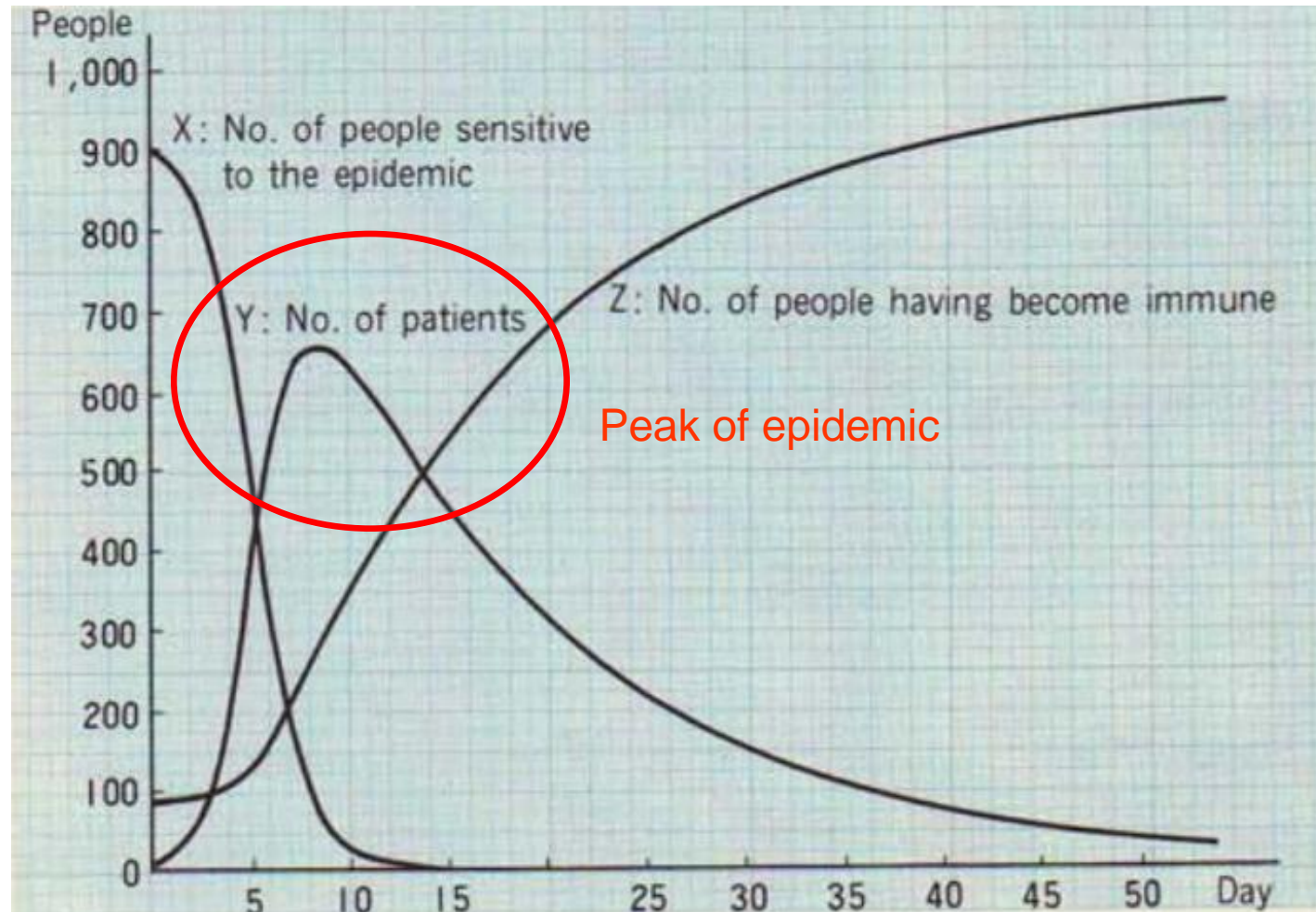
$$z' = 1/14 * y$$

change not yet infected/day

change of sick/day

change of immunized/day

# An “infectious problem” (end)





# Lorenz strange attractor (1)



- In 1963 Edward Lorenz developed a simplified model of the atmospheric convection:

$$x' = a^*(y-x)$$

$$y' = b^*x - y - z$$

$$z' = x*y - c^*z$$

$x(t)$ ,  $y(t)$ ,  $z(t)$  are variables which describe the state of the atmosphere:  
e.g.  $x \sim$  convective movement of air

# Lorenz strange attractor (2)



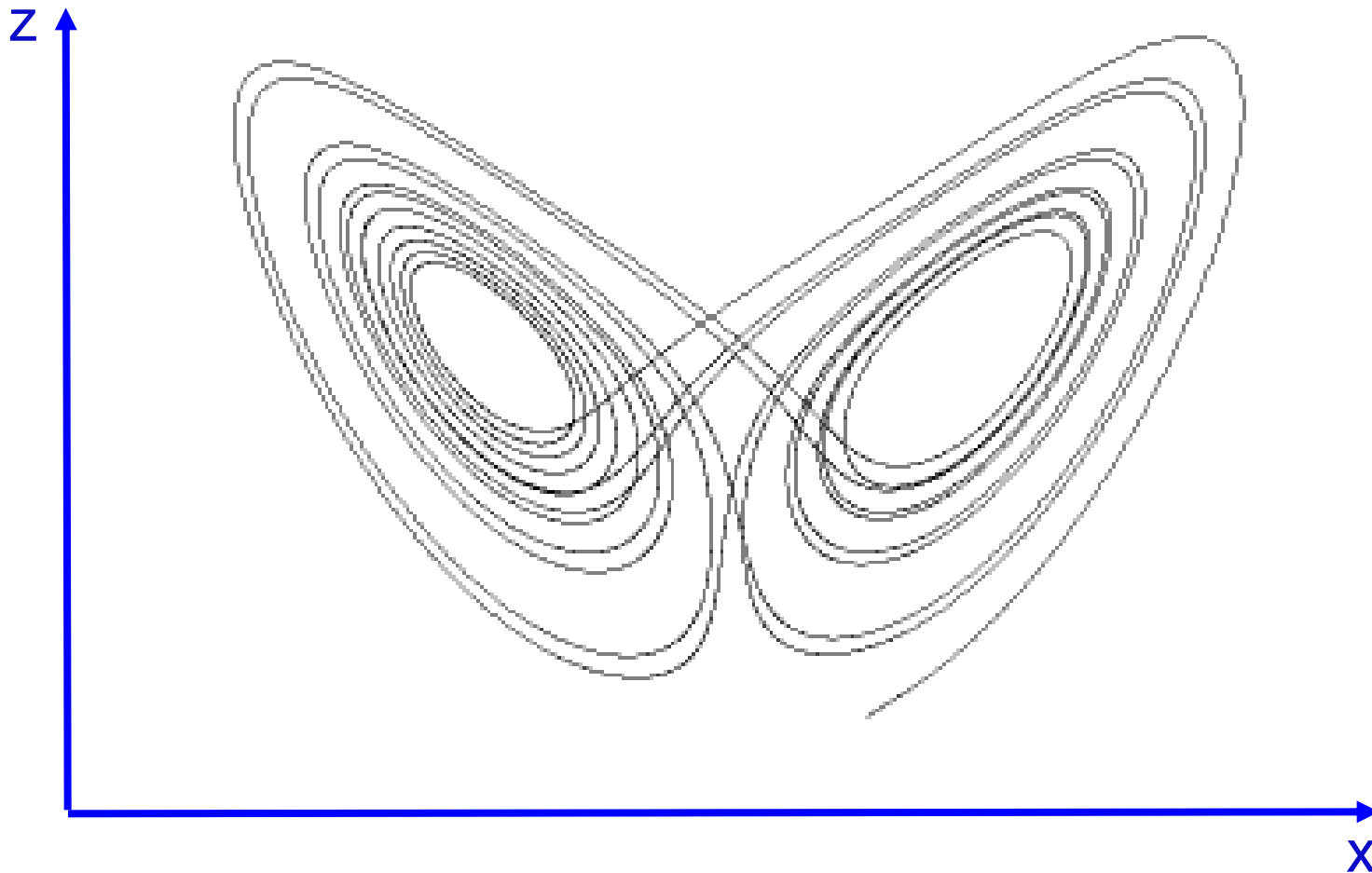
Lorenz found that for the particular values

$$a = 10, b = 8/3 \text{ and } c = 28$$

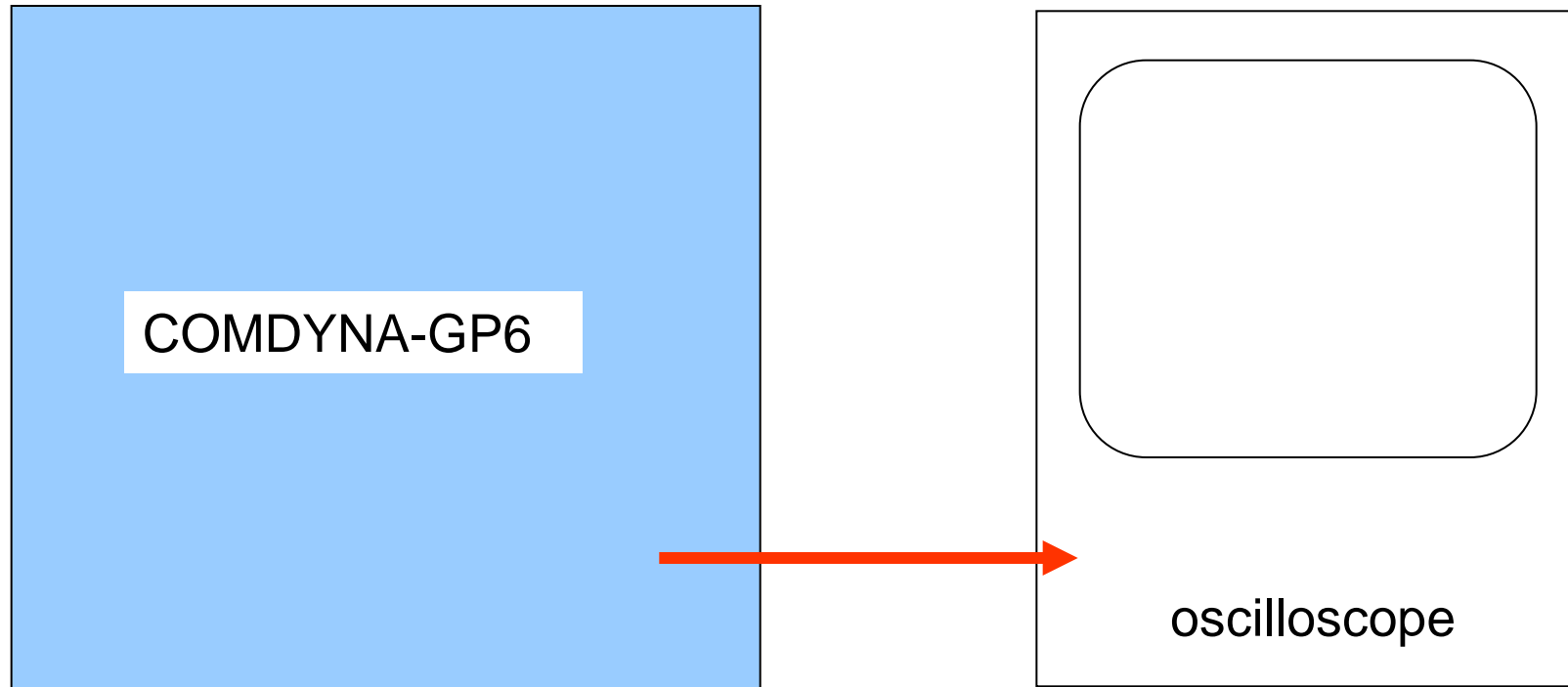
the solutions  $x(t)$ ,  $y(t)$ ,  $z(t)$  become chaotic when the variable time ( $t$ ) cycles through a range of values.

This was the start of the “**chaos theory**” and its related theory of **fractals**.

# Lorenz strange attractor (3)



# Demonstration with COMDYNA-GP6 analog computer

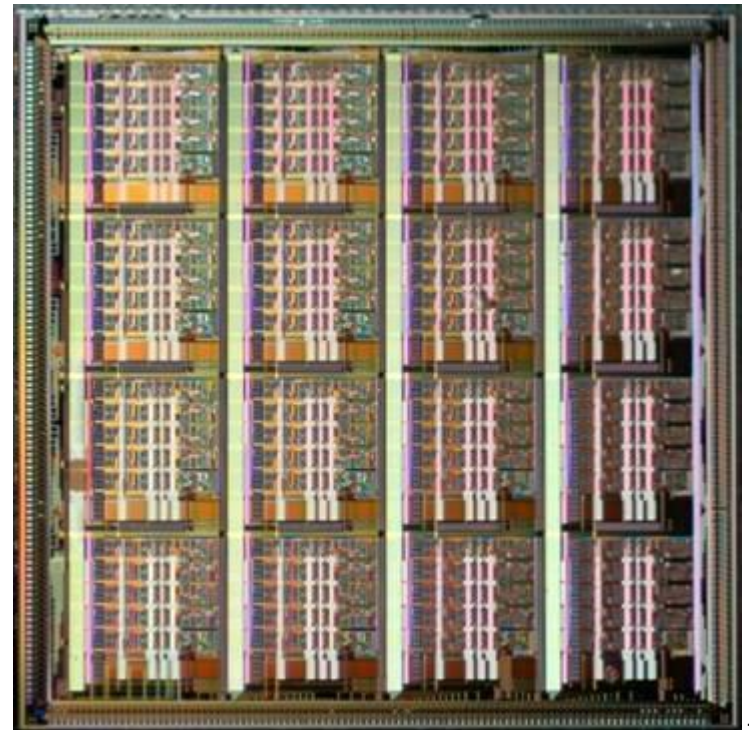


# A possible come-back of the analog computer ? (1)



- G.E.R. Cowan (Concordia University, Montréal) developed in 2005 a single-chip VLSI analog computer ( = a coprocessor) having 80 integrators and 336 other programmable linear and nonlinear circuits.
- The chip can be used to accelerate a digital computer's numerical routines to 23 Gflops\*. The IC is 1 cm<sup>2</sup> and consumes 300 mW, still the lowest energy use of the world.

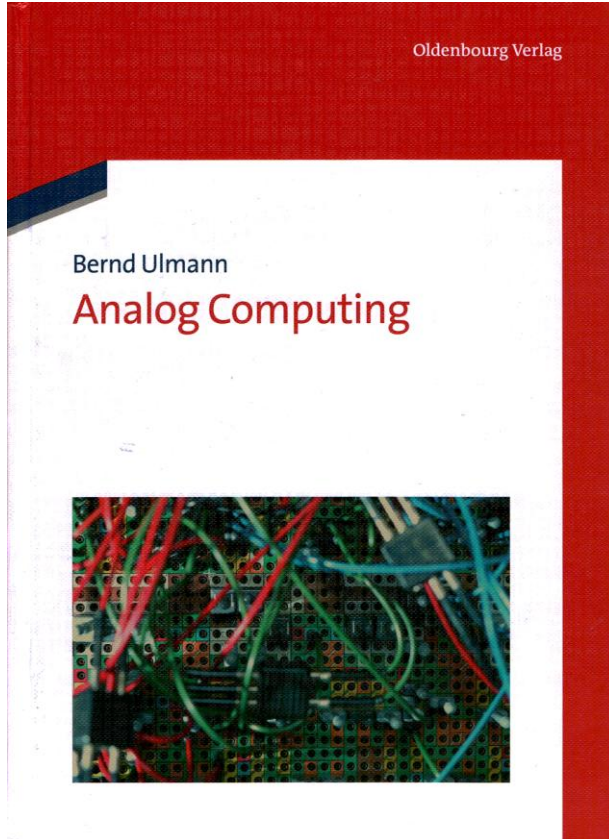
\*[Intel Core-i7: 95 Gflops, 57 W]





78

# Literature



\*Prof. für  
Wirtschaftsinformatik  
FOM, Hochschule f.  
Ökonomie und  
Management,  
Frankfurt/Main

Bernd Ulmann\*: <http://www.analogmuseum.org/>

Joost Rekveld: <http://www.joostrekveld.net/?p=1409> (Analog Art)

<http://computarium.lcd.lu>: Historic Computing Links and Library/PDF's



Lycée classique de Diekirch

# Merci fir d'Nolauschteren!

Slides sinn op

<http://computarium.lcd.lu/news.html>